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A Proof of Goldbach's Conjecture

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Statement of Conjecture

Goldbach's Conjecture, which was announced in 1742, asserts that each even positive integer greater than or equal to 4 is the sum of two prime integers. Thus, e.g., 12 = 5 + 7. Prior to this paper, the Conjecture was still unproved.

First Proof

To prove the Conjecture, we must show that each even positive integer $2k$ is the sum of two odd primes, p, q. I.e., that $2k = p + q$

1. *Definition: diagonal for* $2k$: A diagonal for $2k$ is the set $\{(u, v) \mid u + v = 2k$, where u, v are
odd positive integers $\geq 3\}$. We include (v, u) in the set.
Diagonals for $2k = 8$ through $2k = 22$ are shown in the following lists:

$2k = 8$	$2k = 10$	$2k = 12$
(3, 5)	(3, 7)	(3, 9)
(5, 3)	(5, 5)	(5, 7)
	(7, 3)	(7, 5)
		(9, 3)

$2k = 14$	$2k = 16$
(3, 11)	(3, 13)
(5, 9)	(5, 11)
(7, 7)	(7, 9)
(9, 5)	(9, 7)
(11, 3)	(11, 5)
(13, 3)	

$2k = 18$	$2k = 20$	$2k = 22$
(3, 15)	(3, 17)	(3, 19)
(5, 13)	(5, 15)	(5, 17)
(7, 11)	(7, 13)	(7, 15)
(9, 9)	(9, 11)	(9, 13)
(11, 7)	(11, 9)	(11, 11)
(13, 5)	(13, 7)	(13, 9)
(15, 3)	(15, 5)	(15, 7)
	(17, 3)	(17, 5)
		(19, 3)

Figure 1: Examples of Diagonals

Each ordered pair has a left-hand element and a right-hand element. The set of all left-hand elements is called the *left-hand sequence*, and the set of all right-hand elements is called the *right-hand sequence*

The elements in the left-hand and right-hand sequences are fixed
The elements in a left-hand sequence are a sub-set of the elements of all left-hand sequences that follow in diagonals for larger 2ks, and similarly for the elements in a right-hand sequence.

2. How a diagonal for $2k + 2$ is constructed from a diagonal for $2k$

(A) The left-hand sequence is extended to the next largest odd positive integer after the bot- tom element of the sequence. Thus, in the diagonal for $2k = 18$, the left-hand sequence is extended to 17. This extended sequence now becomes the left-hand sequence of the diagonal for $2k + 2$.

(B) This new left-hand sequence for $2k + 2$ is now turned upside down and becomes the right- hand sequence in the diagonal for $2k + 2$.

(1)
The number of primes in the $2k + 2$ diagonal pairs must be the same as the number of primes in the $2k$ diagonal, or one greater.

3. *Definition:* a *counterexample diagonal*, or just a *counterexample* for short, is a diagonal in which there is no ordered pair (p, q), where p, q are primes.
A noncounterexample diagonal, or just a noncounterexample, is a diagonal in which there is at least one pair (p, q), where p, q are primes. (At the time of this writing, each even positive integer $2k$, where $\{4 \leq 2k \leq (4)(10^{18})\}$, is known, by computer test, to be the sum of two primes, i.e., to be in conformity with Goldbach's Conjecture, and hence not a counterexample.)

4. From "How a diagonal for $2k + 2$ is constructed from a diagonal for $2k$", above, we claim the following:
Let d be any diagonal.
If d is a counterexample, then we denote d by dc.
If d is a noncounterexample, then we denote d by dn. Then it follows from step 2 that $dc = dn$.

This is, of course, absurd, and therefore we conclude that there are no counterexamples, and hence Goldbach's Conjecture is true.

5. Another way of stating the conclusion of step 4 is: *there is one and only one set of diago- nals, whether or not a counterexample exists.* It is important that the reader understand the following distinction: suppose we have a very long sequence of results of flips of a fair coin. The sequence might begin 0, 1, 1, 0, 0, 0, 1, 0, 1, ...

For each $n \geq 1$, there is one and only one nth digit in the sequence. However, that digit could be its "opposite" (where we are considering 1 and 0 to be "opposites").

That kind of thing cannot happen in the case of diagonals. No matter how big $2k$ is, we can describe exactly what the diagonal for $2k$ is. We cannot do the equivalent in the case of the sequence of 1s and 0s.

Second Proof
We show, as in "First Proof", that there is one and only one possibility for each diagonal, whether or not a counterexample exists, which implies (step 4 of "First Proof") that there are no counterexamples.

First we show that there is one and only one possibility for the second element in each ordered pair in a diagonal, whether or not counterexamples exist.
1. *Definition of the "number-slope":*
A number is an odd, positive integer. A number can be a prime, like 5, or a composite, like 9.

A *number-slope* is the set of all occurrences of one number as the right-hand element in ordered pairs in an infinite succession of diagonals for 2k. Thus, in the list of diagonals in Fig. 1, the 3-slope begins:

3 in (5, 3),
3 in (7, 3),
3 in (9, 3),
3 in (11, 3),
3 in (13, 3,)
3 in (15, 3),
3 in (17, 3),
3 in (19, 3),
etc.

The reader can trace other number-slopes in Fig. 1 in "First Proof".

The reason for the slope is steps 2. (A), (B), in "First Proof". The appended odd positive inte- ger becomes the first element in the right-hand sequence in the diagonal for $2k + 2$, and "pushes down" all the elements in what was the left-hand sequence for $2k$.

The number in a given number-slope is *fixed* in each diagonal. It cannot "disappear", "be lost", "move to another cell", "change", etc., in that diagonal. All of which is in keeping with the sentences in step 4 of "First Proof":

Second, we show that there is one and only possibility for the first element in each ordered pair in a diagonal, whether or not counterexamples exist.

Definition of the "Number-Horizontal Line":
A *number-horizontal line* is the set of all occurrences of one number as the *left-hand element* in ordered pairs in an infinite succession of diagonals for $2k$. Thus, in Fig. 1 in "First Proof", the 7-horizontal line begins with the 7 in the ordered pairs

(7, 3), (7, 5), (7, 7), (7, 9), (7, 11), (7, 13), (7, 15), etc.

The reader can trace other number horizontal lines in Fig. 1.
The number in a given number-horizontal line is fixed in each diagonal. It cannot "disappear", "be lost", "move to another cell", "change" in that diagonal, etc.

3. From steps 1 and 2 in this Proof we assert, as we did in "First Proof":
Let d be any diagonal.
If d is a counterexample, then we denote d by dc.
If d is a noncounterexample, then we denote d by dn."
"Then it follows [from steps 1 and 2 in this Proof] that $dc = dn$.
"This is, of course, absurd, and therefore we conclude that there are no counterexamples, and hence Goldbach's Conjecture is true."

Another way of expressing our conclusion is the following:

From step 3 in "First Proof" we have:

Definition: a *counterexample diagonal,* or just a *counterexample* for short, is a diagonal in which there is no ordered pair (p, q), where p, q are primes.

From steps 1 and 2 in this Proof, we assert:
There is one and only one set of diagonals, whether or not a counterexample exists.

But that means there is no difference between a diagonal if it contains an ordered pair (p, q), where p, q are primes, and that same diagonal if it does not contain such an ordered pair.

But that is absurd, "and therefore we conclude that there are no counterexamples, and hence Goldbach's Conjecture is true."

We must not fail to point out that what we have said can be expressed as:

Over the entire infinite sequence of diagonals, each odd positive integer beginning with 3 is the left-hand element in an infinite sequence of pairs containing, as right-hand elements, all odd, positive integers (hence all odd positive primes) beginning with 3.

The reader can see examples of the beginning of some of these infinite sequences of pairs in the example diagonals in step 1 of "First Proof".

Remark
A third proof might be possible by showing that the existence of a counterexample implies a contradiction to the fact (see (1) in step 2 of "First Proof") that each successive diagonal must have the same number of primes as the preceding non-counterexample diagonal, or at most one more than that number ..

For ease of understanding, we will show how the diagonal for the even positive integer $2k = 18$ in step 1 of "First Proof" would have to change in order to become a counterexample diagonal.

1. The pairs in each diagonal are divided into an upper half and a lower half. If the number of pairs is odd, then there is an additional pair between them. This is the case in our example, the additional pair being (9, 9).
   The right-hand elements in the upper pair become, in reversed order, the left-hand elements in the lower pair. Thus, in our example, the right-hand elements 15, 13, 11 in the upper pair, become the elements 11, 13, 15 in the lower half.
2. The total number of primes in our example diagonal before it becomes a counterexample is five, namely 3, 5, 7, 11 and 13. Now in a counterexample, 13 and 11 in the right-hand elements of the upper half would need to become composites in order to eliminate pairs of primes in the diagonal. This change would occur in the left-hand elements in the lower half.

But it would lower the number of primes in the diagonal to three, namely 3, 5, and 7, which is less than the original number, contradicting the rule that in each successive diagonal, the number of primes must be the same as, or one more than, the number in the preceding diagonal (see (1) in step 2 of "First Proof").

If our reasoning is correct, this contradiction would give us a proof of the Conjecture.