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About Fermat's Last Theorem

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ABSTRACT

In this paper, the possibility of finding a simple proof of Fermat's Last Theorem is discussed by using the principles of elementary algebra instead of using the Modularity theorem. For an odd prime n the Fermat's diophantine equation $x^n = y^n + z^n$ gives rise to an equation of $(n - 1)$th degree which can be proved to be irreducible over the field Q of rational real numbers by using Eisenstein's criterion. This proves the theorem for any odd prime n. For $n = 4$ we use a method of *reductio-ad-absurdum* to prove the theorem. Finally, we deduce Beal's conjecture.

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Keywords: Highest Common Factor (HCF), Greatest Common Divisor (GCD), Max / Min(a, b, c) stands for the Largest/Least among the numbers a, b and c, Lowest Common Multiple (LCM), $O(n)$ and $M(n)$ with the usual meanings

Historical Introduction

Pierre de Fermat (20th August 1601–12th January 1665), a Frenchman of Paris had no Mathematics training and he evinced no interest in its study until he was past 30 [1, 2].

To him it was merely a hobby to be cultivated in leisure time. Yet no practitioner of his day made greater discoveries or contributed more to the advancement of Mathematics. By profession he was a lawyer and a politician. His contributions to number theory overshadow all else. Adamantly refusing to bring his work to the state of perfection and their publication, he thwarted the several efforts of others to make the results available in print under his name. Most of what little we know about his investigations, is found in the letters to his friends or notes in the margin of whatever book he happened to be using. This habit of communicating results piece meal, usually as challenges, was particularly annoying to the Parisian Mathematicians. At one point they angrily accused Fermat of posing impossible problems and threatened to break off correspondence unless more details were forthcoming. Because his parliamentary duties demanded an ever-greater portion of his time, Fermat was given to inserting notes on the margins of his personal copy of the Bachet edition of Diophantus-Arithmatica, many of his theorems in number theory. These were discovered five years after his death by his son Samuel, who brought out a new edition of Arithmatica, incorporating his father's celebrated marginalia. By far the most famous is the one written in 1637 in the margin of Arithmatica, which states that: It is impossible to write a cube as a sum of two cubes, a fourth power as a sum of two fourth powers and in general, any power beyond the second, as a sum of two similar powers in non-zero integers. For this, I discovered a truly wonderful proof, but the margin is too small to contain it. The above statement of Fermat is known as Fermat's Last Theorem (hereafter we write in short FLT). Despite efforts of many mathematicians and amateurs, it couldn't be proved for about 350 years. In 1955 Yuataka Taniyama of Japan announced a theory on elliptic curves, which turned out later as a link leading to a proof of FLT. After some hectic research, he published his findings in 1955 along with a conjecture, known as *Yutaka Taniyama Conjecture* (now known as Modularity Theorem). It states that, for every elliptic curve $y^2 = ax^3 + bx + c$ over the rational field $\mathbb{Q}$, there exists non-constant modular functions, $f(z)$ and ϕ (z) such that $f(z)^2 = a\phi(z)^3 + b\phi(z) + c$.

He died in 1958. Goro Shimura, a close friend of Taniyama, tried very hard for about 25 years in search of a proof of this, but could not succeed. Later Kenneth Ribet of USA made intensive research on the conjecture, but could not find the connection between the Taniyama Conjecture and the FLT. But he arrived at the conclusion that – If the Taniyama-Shimura Conjecture is true, then it should imply that the FLT is also true. During the year 1986, Andrew Wiles of Cambridge, UK got the journal in which Ribet's research was published.

On 23rd June 1993, Andrew Wiles announced a proof of FLT, but it had some flaw [2]. When all his efforts to correct the flaw failed, he returned to avail of the assistance of Richard Taylor who was once a student of Andrew Wiles and later his colleague, in research on rectifying the flaw. Together, Andrew Wiles and Richard Taylor published their proof of FLT, for international scrutiny in May 1995. The proof consists of two parts: Modular Elliptic Curves and FLT by Andrew Wiles and Ring Theoretic properties of some Hecke Algebras by Richard Taylor. Wile's proof is based on one significant point in the paper by Richard Taylor. This approach was much simpler and shorter than Wile's original proof of 1993. Still the number of pages is more than 200, whereas the original proof contained about 1000 pages. The Taniyama Conjecture was fully proved by C. Breuil, B. Conrad, F. Diamond and R. Taylor in 1999, based on the Wile's work. Now the conjecture has become a Theorem known as the Modularity Theorem.

Andrew Wiles came to the conclusion that Fermat could never have proved FLT with the limited methods available to him and that Fermat's claim of having a simple proof, was far from truth [4].

When $n=pk$ where p is prime, the Fermat's equation $x^n=y^n+z^n$ becomes $(x^k)^p=(y^k)^p+(z^k)^p$ which is of the form $u^p = v^p + w^p$. If this equation cannot have a non-trivial integer solution, then there will be no solution of the form $u=x^k$, $v=y^k$, $w=z^k$, implying that $x^n=y^n+z^n$ will not have non-zero integer solution. Thus, it is sufficient to prove FLT for $n = 4$ and n = an odd prime. Fermat used his method of 'infinite descent' to prove the impossibility of satisfying $x^4 = y^4 + z^4$. Euler proved FLT for $n = 3$ in 1770 by using the method of `infinite descent'. Kummer proved FLT for all prime between 3 and 100 except 37, 59, 67 which are called irregular primes [4,5].

In this paper, we attempt to present some arguments which is probably the method for the simple proof of FLT, anticipated by Fermat in 1637.

Fermat's Last Theorem

The diophantine equation

$$x^n = y^n + z^n \qquad (1)$$

has no non-trivial integer solution when n is a positive integer greater than 2.

For $n = 1$, it is trivially true, since $(p+q,p,q)$ satisfies (1) where p, q are co-primes.

For $n = 2$ we can rewrite (1) as

$$u^2 - v^2 = 1 \qquad (2)$$

Where $u = \frac{x}{z}$, $v = \frac{y}{z}$. That is,

$$(u-v)(u+v) = 1 \qquad (3)$$

Assuming x,y,z are integers, u and v will be rational numbers so that $u - v$ and $u + v$ are also rational. Therefore, we may take

$u - v = \frac{p}{q}$ and $u + v = \frac{q}{p}$, where p and q are co-primes.

Solving the last two equations we see that

$$\frac{x}{z} = u = \frac{(q^2+p^2)}{2qp} \quad \text{and} \quad \frac{y}{z} = v = \frac{(q^2-p^2)}{2qp} \qquad (4)$$

so that $x = q^2+p^2$, $y=q^2-p^2$ and $z = 2pq$ will satisfy $x^2 = y^2 + z^2$.

As a general statement we can say that $x^2 = y^2 + z^2$ has infinitely many solutions in Pythagorean triples as (i) Integers (ii) Rational Numbers (iii) Real Numbers.

Fermat's Last Theorem for an Odd Prime

From equation (1) we have

$$\left(\frac{x}{z}\right)^n - \left(\frac{y}{z}\right)^n = 1 \qquad (5)$$

or

$$u^n - v^n = 1 \qquad (6)$$

where we define $u = \frac{x}{z}$, $v = \frac{y}{z}$ such that $x - y = hp,\ z = hp$

where h is the HCF of $x–y$ and z and p,q are coprimes. Similarly, we define $u' = \frac{x}{y}, v' = \frac{z}{y}$ such that $x - z = h'p'$, $y = h'q'$

where h' is the HCF of $x - z$ and y and p', q' are coprimes. Factoring (6) we have

$$(u-v)(u^{(n-1)} + u^{(n-2)}v + \cdots + uv^{(n-2)} + v^{(n-1)}) = 1$$

Also

$$u - v = \frac{p}{q} \qquad (7)$$

$$\Rightarrow u^{(n-1)} + u^{(n-2)}v + \cdots + uv^{(n-2)} + v^{(n-1)} = \frac{q}{p} \qquad (8)$$

where $p < q$ since $x^n = y^n + z^n < (y+z)^n$ implies $x<y+z$

Similarly $n' - v' = \frac{p'}{q'}$ where $p' < q'$ and hence

$$u'^{n-1} + u'^{n-2}v' + \cdots + u'v'^{n-2} + v'^{n-1} = \frac{q'}{p'}.$$

The points of intersection of (7) and (8) lie on (6) since the product of LHS of (7) and (8) and the RHS of (7) and (8) satisfy (6)

If $\left(\frac{a}{b}, \frac{c}{d}\right)$ is any positive rational solution of (6) then by choosing $p = ad - bc,\ q = bd$, it is noted that this solution lies in

the solution set of (7) and (8). Hence it is sufficient to solve (7) and (8) instead of (6). The equation of straight lines joining the origin to the points of intersection of (7) and (8) will be of the form

$$u^{(n-1)} + u^{(n-2)}v + \cdots + uv^{(n-2)} + v^{(n-1)} = \frac{q}{p}\left[(u-v)\frac{q}{p}\right]^{(n-1)} = \left(\frac{q}{p}\right)^n (u-v)^{(n-1)} \qquad (9)$$

Since (9) is a homogeneous equation in u, v of degree $n - 1$, it represents $(n- 1)$ straight lines through the origin of uv-plane, which may be real and/or imaginary. The slope $m = \frac{v}{u}$ of the straight line contained in (9) satisfy

$$1 + m + \cdots + m^{n-1} = \left(\frac{q}{p}\right)^n (1-m)^{n-1}$$

That is

$$1 - m^n = \left(\frac{q}{p}\right)^n (1-m)^n \qquad (10)$$

Letting $m = \lambda + 1$ or

$$\lambda = m - 1 = \frac{v}{u} - 1 = \frac{y}{x} - 1 = \left(\frac{-hp}{x}\right)$$

we have

$$(\lambda+1)^n - 1 - \left(\frac{q}{p}\right)^n \lambda^n = 0$$

$$\text{i.e. } \binom{n}{1}\lambda + \binom{n}{2}\lambda^2 + \cdots + \binom{n}{n-1}\lambda^{n-1} + \left[1 - \frac{q^n}{p^n}\right]\lambda^n = 0$$

$$i.e.\ p^n\left[\binom{n}{1}+\binom{n}{2}\lambda+\cdots+\binom{n}{n-1}\lambda^{(n-2)}\right]-(q^n-p^n)\lambda^{(n-1)}=0 \quad (11)$$

$$i.e.\ xp\left[\binom{n}{1}x^{n-2}-\binom{n}{2}x^{n-3}(hp)+\cdots+\binom{n}{n-1}(hp)^{(n-2)}\right]$$

$$=(q^n-p^n)h^{(n-1)}$$

$$\binom{n}{1}x^{n-1}(hp)-\binom{n}{2}x^{n-2}(hp)^2+\cdots-\binom{n}{n-1}x(hp)^{n-1}$$

$$=(hq)^n-(hp)^n \quad (12)$$

Similarly

$$\binom{n}{1}x^{n-1}(h'p')-\binom{n}{2}x^{n-2}(h'p')^2+\cdots-\binom{n}{n-1}x(h'p')^{n-1}$$

$$=(h'q')^n-(h'p')^n \quad (13)$$

It is clear that (12) and (13) are equivalent to (1) in disguise, but $nx(x-hp)(hp)$ is a factor of LHS of (12) and $nx(x-h'p')(h'p')$ is a factor of LHS of (13). Since $x^n=y^n+z^n$ implies $x\equiv y+z \bmod n$ by Fermat's little theorem, we have

$$h(q-p)\equiv 0\equiv h'(q'-p') \bmod n$$

These conditions give rise to four possibilities:
(i) $h\equiv 0 \bmod n$
(ii) $h'\equiv 0 \bmod n$
(iii) $q\equiv p \bmod n$ and
(iv) $q'\equiv p' \bmod n$

First, we shall show that (i) and (ii) are false. In order to prove the falsity of (ii) we show that (i) $h'\equiv 0\equiv h \bmod n$ and (ii) $h'\equiv 0\equiv (q-p) \bmod n$ are false. Then by using the principle of symbolic logic in the form, if A, B, C are logical statement satisfying the conditions (i) $T(B\vee C)=1$ (ii) $T(A\wedge B)=0=T(A\wedge C)$

then $T(A)=0$ or A is false. This follows from the fact that

$$T[A\wedge(B\vee C)]=T[(A\wedge B)\vee(A\wedge C)]\le T(A\wedge B)+T(A\wedge C)=0+0$$

So that $T[A\wedge(B\vee C)]=0$ therefore $T(A)=0$ where the truth value function T is non-negative. It is possible to define $A\equiv(h'\equiv 0 \bmod n)$, $B\equiv(h\equiv 0 \bmod n)$ and $C\equiv(q\equiv p \bmod n)$ so that $B\vee C\equiv(h(q-p)\equiv 0 \bmod n)$ which has truth value 1, by the assumption that there exists a positive integral solution of equation (1).

According to the possibility (i) $h'\equiv 0\equiv h \bmod n$ we have $x-y\equiv 0\equiv z \bmod n$ and $x-z\equiv 0\equiv y \bmod n$ implying that $x\equiv y\equiv z\equiv 0 \bmod n$ so that n is a common factor of x, y, z which contradicts the assumption that GCD $(x, y, z)=1$. This contradiction proves that the possibility (i) $h'\equiv 0\equiv h \bmod n$ is false. According to the possibility (ii) $h'\equiv 0\equiv(q-p) \bmod n$ we have $x-z\equiv 0\equiv y \bmod n$ and $q=p+kn$ for some positive integer k so that $h'p'=M(n)$ and $h'q'=M(n)$ and hence the RHS of (13) is $M(n^n)$ and on LHS we have the first term $M(n^2)$ unless $x\equiv 0 \bmod n$ but the remaining terms are $M(n^3)$ atleast, due to the presence of $(h'p')$.

The LHS of (13) consists of terms of order $M(n^2)$, $M(n^3)$ etc. due to the presence of $h'p'$ so that all terms except the first term are divisible by n^3 and RHS is also divisible by n^3, since RHS $=(hq)^n-(hp)^n=M(n^n)$ is divisible by n^3, but the first term of LHS is not divisible by n^3 unless $x=M(n)$. This disparity implies that we must have $x=M(n)$ in addition to $y=h'q'=M(n)\Rightarrow x, y, z$ are $M(n)$.

This contradiction proves that the possibility (ii) $h'\equiv 0\equiv(q-p) \bmod n$ is false. Since both possibilities (i) and (ii) are false, it follows that $h'\equiv 0 \bmod n$ is false, by the principle of symbolic logic stated earlier. Similarly, $h\equiv 0 \bmod n$ is also false. $\therefore h'\not\equiv 0\not\equiv h \bmod n$. Now we are left with the possibilities (iii) $q\equiv p \bmod n$ and (iv) $q'\equiv p' \bmod n$ or equivalently

$$q=p+Kn \quad (14)$$

and

$$q'=p'+K'n \quad (15)$$

for some positive integers K and K'. It will be shown that (14) and (15) are false statements. (14) implies

$$q^n-p^n=(p+Kn)^n-p^n=\sum_{r=0}^{n-1}\binom{n}{r}p^r(Kn)^{(n-r)}$$

is divisible by n^2 at least, since n divides $\binom{n}{r}$ for $0<r<n$ and $n-r>2$ for $r=0$. $\Rightarrow (hq)^n-(hp)^n$ is divisible by n^2.

Next, we claim that $p\equiv 0 \bmod n$ and $q\equiv 0 \bmod n$ are false for the following reasons (i) $p\equiv 0\equiv q \bmod n$ implies that n is a common factor of p and q which are coprimes (ii) $p\equiv 0\not\equiv q \bmod n$ implies that $x\equiv y \bmod n$ but $z\not\equiv 0 \bmod n$. This is absurd since $x\equiv y \bmod n\Longrightarrow z\equiv 0 \bmod n$. (iii) $q\equiv 0\not\equiv p \bmod n$ implies that $z\equiv 0 \bmod n$ but $x\not\equiv y \bmod n$ which is false since $z\equiv 0 \bmod n$ implies $x\equiv y \bmod n$. Hence p, q are not divisible by n. Similarly, p', q' are not divisible by n and so also are $y=h'q'$ and $z=hq$.

Rewriting equation (12) we have

$$\left[\binom{n}{1}x^{n-1}(hp)-\binom{n}{n-1}x(hp)^{n-1}\right]-\left[\binom{n}{2}x^{n-2}(hp)^2-\binom{n}{n-2}x^2(hp)^{n-2}\right]+\cdots=(hq)^n-(hp)^n \quad (16)$$

On the LHS of equation (16) the first bracket is divisible by $\binom{n}{1}$ so that the first bracket is of order M (n) and remaining brackets on LHS are of order at least O (n^2) but all these brackets turn out to be $M(n^2)$ by choosing $x(hp)=M(n)$ but the RHS is divisible by n^2. This disparity demands us to let $x(hp)=M(n)\Longrightarrow x=M(n)$. Since $h\neq M(n)$ and $p\neq M(n)$.

If $x=M(n)$ then $y+z=M(n)=\bar{K}n$ for some positive integer $\bar{K}$.

$$\therefore (y+z)^n-(y^n+z^n)=\sum_{r=1}^{n-1}\binom{n}{r}y^r z^{(n-r)}\Rightarrow$$

$\therefore M(n^n)=yzO(n^t)$ where $t=\frac{n-1}{2}$ so that $yz=O(n^{t+1})=O(n)$ at least $=M(n)$ at least since yz is a positive integer. This is also clear from the fact that n divides $\binom{n}{r}$

for $0 < r < n$ but $y^{r-1}z^{n-r-1}$ of degree $n-2$ cannot give rise to a factor of order

$M(n^{n-1})$ or more

since $y^n + z^n = x^n = M(n^n)$ and $(y+z)^n = M(n^n)$

$\therefore yz = M(n)$

which contradicts the earlier assertions $h \not\equiv 0 \not\equiv h'$ mod n, $q \not\equiv 0 \not\equiv q'$ mod n and $y = h'q' \neq M(n)$, $z = hq \neq M(n)$. This implies equations (14) and (15) are false. Hence the possibilities (iii) and (iv) stated earlier are false. Thus all the four possibilities (i) to (iv) stated earlier are false. Hence $q - p$ and $q' - p'$ are not divisible by n$\Rightarrow$ $q^n - p^n = q - p + M(n)$ and $q'^n - p'^n = (q' - p') + M(n)$ are not divisible by n.

Hence the polynomial in λ of equation (11) satisfies Eisenstein's criterion [1, 3] for irreducibility over $\mathbb{Q}$ since $\binom{n}{1}, \binom{n}{2}, \ldots, \binom{n}{n-1}$ are divisible by n but $n \nmid p^n$, $n \nmid (q^n - p^n)$ and $n^2 \nmid \binom{n}{1}p^n$.

Therefore, the roots of (11) are irrational. Hence λ and $m = \lambda + 1$ can have only irrational values $\Rightarrow$ Equations (7) and (8) and hence (6) cannot have a positive rational solution. This proves that equation (1) has no positive integral solution. Hence FLT is true for any odd prime n. This is also clear from the fact that we have shown the falsity of possibilities (i) to (iv) stated earlier.

Fermat's Last Theorem for $n = 4$

By letting $n = 4$ in equations (5) to (10), the equation (10) becomes [5]

$$1 - m^4 = \frac{q^4}{p^4}(1-m)^4$$

Letting $m = \lambda + 1$ or $\lambda = m - 1 = \left(\frac{-hp}{x}\right)$ we have

$$(\lambda+1)^4 - 1 + \frac{q^4}{p^4}\lambda^4 = 0 \Rightarrow (1 + \frac{q^4}{p^4})\lambda^4 + 4\lambda^2 + 6\lambda^2 + 4\lambda = 0$$

$$\left(p^4 + q^4\right)\lambda^3 + 2p^4\left(2\lambda^2 + 5\lambda + 2\right) = 0 \qquad (17)$$

$$\left(p^4 + q^4\right)\left(\frac{-h^3p^3}{x^3}\right) + 2p^4\left[2\left(\frac{h^2p^2}{x^2}\right) - 3\left(\frac{hp}{x}\right) + 2\right]$$

$$\therefore \frac{\left(p^4 + q^4\right)h^3}{p} = 2x\left[2h^2p^2 - 3hpx + 2x^2\right] \qquad (18)$$

Similarly

$$\frac{\left(p'^4 + q'^4\right)}{p'}h'^3 = 2x\left[2h'^2p'^2 - 3h'p'x + 2x^2\right] \qquad (19)$$

From $x^4 = y^4 + z^4$ we have $x \equiv y + z$ mod 2 so that $x - y \equiv z$ mod 2 and $x - z \equiv y$ mod 2 or $h(q - p) \equiv 0 \equiv h'(q' - p')$ mod 2. In order to prove the falsity of $h' \equiv 0$ mod 2, we consider the possibilities (i) $h' \equiv 0 \equiv h$ mod 2 and (ii) $h' \equiv 0 \equiv (q - p)$ mod 2 and prove that these conditions are invalid as in Section **'Fermat's Last Theorem for an Odd Prime'.**

In the former possibility, we have $x - z \equiv 0 \equiv y$ mod 2 and $x - y \equiv 0 \equiv z$ mod 2 implying that x, y, z are even positive integers. This contradiction proves that possibilities (i) is invalid. In the possibility (ii) we have $h' \equiv 0$ mod 2 and $q \equiv p$ mod 2 so that h' is even and p, q are odd positive integers, since they are coprimes. Also $y = h'q'$ is even and hence x, z are odd $h'p'$ is also even. If $y = h'q'$ is even then we may write $y = 2y_1$, $x - z = 2k_1y_1$, $x + z = 2k_2y_1$ where k_1 and k_2 are positive integers so that $x^2 + z^2 = 2(k_1^2 + k_2^2)y_1^2$ and $y^4 = x^4 - z^4$ implies $16y_1^4 = 2k_1y_1.2k_2y_1.2(k_1^2 + k_2^2) = 8k_1k_2(k_1^2 + k_2^2)y_1^4$. That is $k_1k_2(k_1^2 + k_2^2) = 2$. This has solution $k_1 = k_2 = 1$ only leading to $z = 0$ and $x = y$, the trivial solution for FLT. This proves that y cannot be even. $\Rightarrow h'$ and q' must be odd. If p' is even, then $h'p' = x - z$ is even $\Rightarrow y = x - z + M(2) = M(2)$ which is already ruled out above. Hence h', p', q' are all odd positive integers. Therefore the statement $h' \equiv 0$ mod 2 and $q \equiv p$ mod 2 is false. Similarly h, p, q are all odd and statement $h \equiv 0$ mod 2 and $q' \equiv p'$ mod 2 is false.

Hence $y = hq$, $z = h'q'$ are odd integers and hence x must be an even positive integer. We shall show that this statement is invalid. Suppose $x^4 = y^4 + z^4$ where x,y,z are positive integers such that their GCD = 1. Letting $X = x^2$, $Y = y^2$, $Z = z^2$ we have $X^2 = Y^2 + Z^2$ so that (X, Y, Z) from a Pythagorean triple with solution

$$X = Q^2 + P^2, Y = Q^2 - P^2, Z = 2QP$$

where P, Q are coprimes (with $Q > P$) in which Y and Z can be exchanged due to symmetry. Clearly Z/Y is even so that X must be an odd integer.

$\therefore x^2$ and hence x must be an odd integer. Hence the requirement that x is even and y, z is odd, cannot be satisfied. Also, y/z is even implying q/q' is even contradicting the earlier assertion that q and q' are odd. These contradictions prove that $x^4 = y^4 + z^4$ has no positive integral solution i.e. FLT is true for $n = 4'$.

The method of simple proof of FLT might be the method explained in the above Sections **'Fermat's Last Theorem for an odd prime'** and **'Fermat's Last Theorem for $n = 4$'**.

Comparison between Beal's Conjecture and FLT

Inspired by the FLT, Andrew Beal, a banker from Texas, USA [4,5] proposed the following conjecture: If $x^a = y^b + z^c$ where a, b, c are positive integers and may be different as well as greater than 2 and x, y, z are positive integers, have solutions then x, y, z have a common factor greater than 2. The dissimilarity between FLT and Beal's equation is that in FLT we consider values of x, y, z such that any two of them must be co-primes, whereas in Beal's equation, no two of them are co-primes but all of them have a common factor greater than 2.

Without loss of generality, we assume Max$(a,b,c) > 2$ and there is an initial solution of Beal's equation without a common factor of x, y, z. Let L = LCM(a,b,c) so that there exist positive integers a', b', c' such that $L = aa' = bb' = cc'$. Choose m = Min (a', b', c'). Multiplying the initial Beal's equation by p^L where p is any odd prime or an even integer from the set {2, 4, 8, 16, 32,…} if $m \geq 2$ or from the set {4, 8, 16, 32,…} if $m = 1$. We note that p^m will be a common factor of the new x, y, z since $p^L x^a = (p^{a'}x)^a$, $p^L y^b = (p^{b'}y)^b$ and $p^L z^c = (p^{c'}z)^c$ implies that p^m is common for the new values of x, y, z. Since L has either an odd prime or 4 as a factor when there will be at least one choice of p such that p^m is a common factor of new x, y, z after multiplication of the initial Beal's equation by p^L. As examples, consider

$$3^2 = 2^3 + 1^3 \qquad (i)$$

and

$$5^3 = 11^2 + 2^2 \qquad (ii)$$

Multiplying these by 3^6 the results are $81^2 = 18^3 + 9^3$ and $45^3 = 297^2 + 54^2$ so that the common factor is 9. Multiplying (i) and (ii) by 2^6 the results are $24^2 = 8^3 + 4^3$ and $20^3 = 88^2 + 16^2$ so that the common factor is 4. Hence Beal's conjecture is true in general since $\text{Max}(a,b,c) > 2$ and p is chosen as described above.

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