ISSN: 2754-4753

Journal of Physics & Optics Sciences

Review Article

Open Access

About Newton's Law of Gravitation and the Energy Momentum Relations

Chandramohanan MR

Former Professor of Mathematics, Narayanaguru College of Engineering, Manjalummood, Tamil Nadu, India

ABSTRACT

Newtonian dynamics was based on the assumption of instantaneous propagation of interaction. Roamer's studies showed that the velocity of interaction cannot exceed the velocity of light in free space. (c=3x10^8 mps). Lorentz' electron theory and electrodynamics take this fact into account. It is possible to rectify the defects of Newtonian theory by deriving Maxwell-Lorentz equations and the Lorentz Force equation for the gravitational field and hence the modified Newtonian dynamics can explain

(i) The Perihelion Shift of the orbit of the Planet Mercury

(ii) Deflection of light by the Sun

(iii) The Gravitational Redshift [1].

It is shown that the use of centre of mass coordinate system also implies a non-elliptic orbit in general, so that the inverse square law of Newton/Coulomb is always valid. Next, the energy and momentum of a dynamical system are shown to satisfy

$$\bar{E}\Psi = mc^2\Psi - i\bar{h}\left(\frac{\partial}{\partial t}\right)\Psi$$

$$\bar{P}\Psi = m\bar{v}\Psi + i\bar{h}\left(\frac{\partial}{\partial r}\right)\Psi$$

Finally, the Schrodinger's wave function is interpreted as a quantum mechanical energy density [2].

***Corresponding author**

Chandramohanan MR, Former Professor of Mathematics, Narayanaguru College of Engineering, 41/2674 (1) Sreenilayam, Manacaud PO, Thiruvananthapuram, Kerala, India.

Received: May 08, 2024; **Accepted:** May 15, 2024; **Published:** May 24, 2024

Keywords: Notations of Classical Dynamics and Electrodynamics

Introduction

It is a matter of great difficulty to discover and to distinguish the true motion of particular bodies from the apparent, because the parts of the immovable space to which those motions are performed, do by no means come under the observation of our senses we are confronted with the question [3]. How can one give a meaning to the concept of velocity, if you do not have a space to refer? At the same time Newton recognized clearly that only relative quantities could be directly measured. Unlike his relationist contemporaries Huygens, Leibnitz etc., he was convinced that a scientifically useful notion of motion could not be based on relational quantities. Instead, he sought to demonstrate how absolute quantities could be deduced from relative observations. This was severely criticized by Leibnitz who argued that there is no philosophical need for any conception of space apart from the relations of matter and objects. None of the high-minded metaphysics had led to any idea about how to develop a dynamical theory that might challenge the Newtonian theory until the advent of electromagnetic theory of radiation. Before Maxwell, it was supposed that all laws of physics are invariant under the Galilean transformations. But the electro-magnetic theory is in apparent disagreement with the Galilean relativity and the Galilean transformations. To remove this apparent disagreement, H.A. Lorentz introduced a new transformation known as the Lorentz transformation

$$Lx = x' + v_0 t', \qquad y = y'$$

$$Lt = t' + \frac{v_0 x'}{c^2}, \qquad z = z', \qquad L = \sqrt{1 - \frac{v_0^2}{c^2}}$$

which become the Galilean transformation

$$x = x' + v_0 t, \qquad y = y', \qquad z = z' \text{ and } t = t'$$

in the limiting case as $c \to \infty$

In 1887, Voigt had derived a transformation relative to which the formula of Doppler Shift is invariant. This coincides with the LT. The absence of c marks the apparent failure of Newtonian dynamics (along with Galilean transformation) to explain the following

(i) The perihelion shifts of the planet Mercury

(ii) Deflection of light by the Sun

(iii) Gravitational red-shift

But it is possible to derive Maxwell-Lorentz equations for the gravitational fields as well and hence the apparent failures of the earlier Newtonian dynamics, can be removed [1].

A good estimate for the universal time-in theory is given by the proper time interval $d\tau = \sqrt{1-\frac{v_0^2}{c^2}}dt$ and the space-interval is $dx_q = \frac{dx}{\sqrt{1-\frac{v_0^2}{c^2}}}$ so that $dx\,dt = dx_\tau\,d\tau$ [4,5]. This demonstrates the truth of validity of Newtonian logic to deduce absolute quantities from relative observations and this serves as a blow to the criticism of Leibnitz.

Removable Singularity of the Potential Function

Next, we will resolve the singularity of the potential function representing potential energy per unit mass/unit charge. The potential due to a spherical point mass M at a distance r from its centre of mass is given by $\phi_1 = \mathrm{MG}/r$. Consider the potential at $\boldsymbol{r}$ due to the presence of two masses m_1 and m_2 as given by

$$\phi_{12} = \frac{m_1}{|\boldsymbol{r}-\boldsymbol{r}_1|} + \frac{m_2}{|\boldsymbol{r}-\boldsymbol{r}_2|} \qquad (1)$$

where we took G = 1, $\boldsymbol{r}_1$ and r_2, the locations of m_1 and m_2. For convenience, we may take y=0, z=0, y_1=0, z_1=0 and $x_1 = a$, x_2 = b

$$\therefore \phi_{12} = \frac{m_1}{x-a} + \frac{m_2}{x-b} \qquad (2)$$

Let $g = \frac{m_1 b + m_2 a}{m_1 + m_2}$ and $g' = \frac{m_1 a + m_2 b}{m_1 + m_2}$

$$\therefore \phi_{12} = \frac{(m_1+m_2)x - (m_1 b + m_2 a)}{(x-a)(x-b)}$$

$$\therefore \phi_{12} = \frac{(m_1+m_2)(x-g)}{(x-g)(x-g') - \mathrm{A}^2} \qquad (3)$$

where $\mathrm{A}^2 = \frac{m_1 m_2\,(b-a)^2}{(m_1+m_2)^2}$

equation (3) shows that the potential due to two masses will vanish at the centre of mass $x = g$. The conclusions are that: (i) for any system of particles, the origin of potential (zero potential) should be at the centre of mass; (ii) for a single point mass M the potential at its centre of mass must be zero. This assertion demands that the potential function for a single mass M must be chosen in the form $\phi_1 = \frac{[\mathrm{M}]\mathrm{G}}{|\boldsymbol{r}-\boldsymbol{r}_1|}$ when $|\boldsymbol{r}-\boldsymbol{r}_1| = d_1 \leq$radius of the spherical mass, where [M] is the effective mass within the spherical region, *i.e.* within $|\boldsymbol{r}-\boldsymbol{r}_1| = d_1$. Outside this region we may take $\phi_1 = \frac{\mathrm{MG}}{|\boldsymbol{r}-\boldsymbol{r}_1|}$ as usual. Similarly for the potential due to a charge Q, we define $\phi_2 = \left(\frac{Q}{m}\right)\frac{[m]}{4\pi\varepsilon_2|\boldsymbol{r}-\boldsymbol{r}_1|}$, when $|\boldsymbol{r}-\boldsymbol{r}_1| = d_1 \leq$ radius of the spherical charge. Outside this region we may take $\phi_2 = \frac{Q}{4\pi\varepsilon_2|\boldsymbol{r}-\boldsymbol{r}_1|}$, $|\boldsymbol{r}-\boldsymbol{r}_1| >$ radius of the electron, m being the mass of the electron.

Clearly both ϕ_1 and ϕ_2 tend to zero when the radius of [M] and [Q] tend to zero.

Centre of Mass of Many-Particle System

Since gravitational field intensity is not like tension in a string, it can be considered as force per unit area. Therefore, instead of a single line of force field connecting m_1 and m_2, we consider a 'tube' of field lines consisting of the same lines joining these masses. This may be called a 'tube' of field lines. The shortest tube joining m_1 and m_2 will contain the centre of mass G_1 of m_1 and m_2. For a system of n particles, the centre of mass always moves with constant velocity so that the sum of forces of interaction among them is zero. For the study of motion of m_3 in the field of m_1 and m_2 we consider m_3 as one system and $\{m_1, m_2\}$as another system with centre of mass at G_1. The motion of m_3 depends on the tubes of field lines emanating from $\mathrm{G}_1$and terminating at m_3. The shortest tube connecting G_1 and m_3 will contain the overall centre of mass G_2 of the whole system $\{m_1, m_2, m_3\}$. Hence the motion of each of m_1, m_2 and m_3 can be studied by considering the tubes of field lines connecting each particle to the centre of mass of the sub-system not including the particle under study. Hence, we conclude that the potential should be redefined by considering centre of mass as the zero-point of potential.

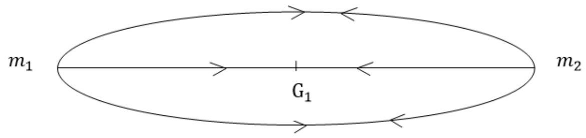

Figure 1

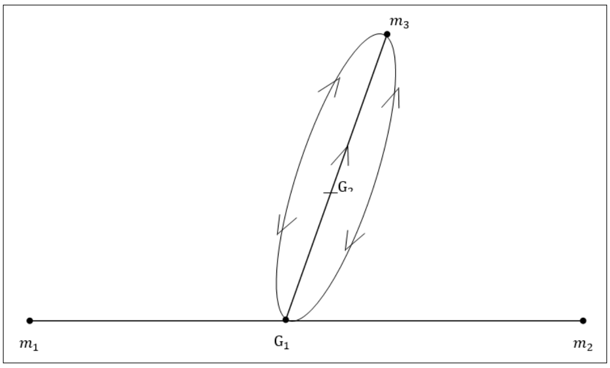

Figure 2

For each body of an n-body problem we define the reduced mass for m_1 by $\mu_{m_1} = \frac{m_1(M-m_1)}{M}$ for m_2 by $\mu_{m_2} = \frac{m_2(M-m_2)}{M}$ etc where $M = \sum_1^n m_i$

We further assume that

(i) Each body m_i is acted upon by a central force towards the centre of mass of the sub system consisting of m_1, m_2,.... m_n excluding the test mass m_i

(ii) The potential at m_1 is $\frac{\mu_{m_1}^2}{m_1 R_1}$ at m_2 is $\frac{\mu_{m_2}^2}{m_2 R_2}$ etc. where $\overline{R}_i$ is the position vector of m_i relative to the center of mass $= \frac{m_1\overline{r}_1 + m_2\overline{r}_2 + \ldots + m_n\overline{r}_n}{m_1 + m_2 + \ldots + m_n}$ and $\overline{r}_1, \overline{r}_2, \ldots$ being the locations of m_1, m_2,..., m_n.

Derivation of Inverse Square Law (by The Principle of Work-Energy)

$$W = \int_A^B \bar{F} \cdot d\bar{r} = \int_A^B \left(\frac{d\bar{p}}{dt}\right) \cdot d\bar{r} \text{(From Newton's Law of Motion)}$$

$$\text{RHS} = \int_A^B \bar{v}.d\bar{p} = [\bar{p}.\bar{v}]_A^B - \int_A^B \bar{p}.d\bar{v}$$

$$= [mv^2]_A^B - \int_A^B mvdv$$

$$= [mv^2]_A^B - \int_A^B m_0 c^2 e^{\frac{v^2}{2c^2}} \left(\frac{v}{c^2} dv\right)$$

$$= \left(mv_B^2 - mv_A^2\right) - \left(m_0 c^2 e^{\frac{v^2}{2c^2}}\right)_A^B = \left[mv^2 - mc^2\right]_A^B$$

$$\text{i.e.RHS} = \left[-mc^2\left(1-\frac{v^2}{e^2}\right)\right]_A^B$$

$$= \left[-m_0 c^2 .e^{\frac{v^2}{2c^2}}\left(1-\frac{v^2}{e^2}\right)\right]_A^B$$

$$= \left[-m_0 c^2\left(1+\frac{v^2}{2c^2}\right)\left(1-\frac{v^2}{e^2}\right)\right]_A^B \text{ since } m = m_0 e^{\frac{v^2}{2c^2}}$$

$$= \left[-m_0 c^2\left(1-\frac{v^2}{2c^2}\right)\right]_A^B = \frac{1}{2} m_0\left(v_B^2 - v_A^2\right) \quad (4)$$

Since the force-field is radial we may take $\overline{F} = f(r)\overline{I}$ where $\overline{I}$ is a unit vector along the radial line from the centre of mass of the subsystem (excluding the test mass m) and hence $\overline{I}$ may be considered as a vector joining the center of mass to the test mass *m*.

$$\therefore F.dr = f(r)dr$$

When *A* and *B* are nearby points in the trajectory, the force may be approximated by a constant value f_0 throughout in the line segment joining *A* and *B*.

$$\therefore \quad \text{LHS} = \int_A^B \overline{F}.d\overline{r} = \int_A^B f_0 dr = f_0\left(r_B - r_A\right) \quad (5)$$

Hence, we have $\frac{1}{2} m_0\left(v_B^2 - v_A^2\right) = f_0\left(r_B - r_A\right)$

$$i.e. \frac{1}{2} m_0\left(v^2 - v_0^2\right) = f_0\left(r - r_0\right)$$

$$\therefore v^2 = v_0^2 + \left(\frac{2f_0}{m_0}\right)\left(r - r_0\right) = f_1^2\left(r - r_1\right)\text{say}$$

$$v = f_1\sqrt{r - r_1}$$

By Fermat's principle of stationary time

$$\delta\int_\alpha^B \frac{ds}{v} = 0 \quad (6)$$

Where $v = f_1\sqrt{r - r_1}$

$$\therefore \delta\left[\int_A^B \left(1 + r^2\theta'^2\right)^{\frac{1}{2}}\left(r - r_1\right)^{-\frac{1}{2}} dr\right] = 0$$

Taking $F = \left(1 + r^2\theta'^2\right)^{\frac{1}{2}}\left(r - r_1\right)^{-\frac{1}{2}}$, the Euler-Lagrange equation [6]

$$\left(\frac{d}{dr}\right)\left(\frac{\partial F}{\partial\theta'}\right) - \frac{\partial f}{\partial\theta'} = 0$$

gives

$$\frac{d}{dr}\left(\frac{\partial F}{\partial\theta'}\right) = 0$$

so that $\frac{\partial F}{\partial\theta'}$ is a constant

$$\therefore \sqrt{\left(r^2\theta'\right)^2 / \left[\left(1 + r^2\theta'^2\right)\left(r - r_1\right)\right]} = A$$

$$\therefore r^4\theta'^2 = A^2\left(1 + r^2\theta'^2\right)\left(r - r_1\right)$$

$$\theta'^2\left[r^4 - A^2 r^2\left(r - r_1\right)\right] = A^2\left(r - r_1\right)$$

$$\therefore r^2\theta'^2 = \frac{A^2\left(r - r_1\right)}{\left[r^2 - A^2\left(r - r_1\right)\right]}$$

$$\frac{1}{r^2\theta'^2} = \frac{r^2 - A^2\left(r - r_1\right)}{A^2\left(r - r_1\right)} = \frac{r^2}{A^2\left(r - r_1\right)} - 1$$

$$\text{Let } u = \frac{1}{r} \Rightarrow \frac{du}{d\theta} = \frac{-1}{r^2}\frac{dr}{d\theta} = \frac{-1}{r^2\theta'}$$

$$\therefore \frac{1}{\left(r^2\theta'^2\right)} = \frac{1}{u^2}\left(\frac{du}{d\theta}\right)^2 = \frac{r^2}{A^2\left(r - r_1\right)} - 1$$

$$i.e. \left(\frac{du}{d\theta}\right)^2 + u^2 = \frac{1}{A^2\left(r - r_1\right)}$$

Differentiating wrt *θ* and cancelling $\frac{2du}{d\theta}$ throughout

$$\Rightarrow \frac{d^2u}{a\theta^2} + u = \frac{1}{2A^2\left(r - r_1\right)^2 u^2} = \frac{MG}{h^2}\frac{r^2}{\left(r - r_1\right)^2} \quad 7\ (7)$$

for some *h*. When $r_1 = 0$ this represents the conic

$$\frac{d^2u}{a\theta^2} + u = \frac{MG}{h^2}$$

If *f*(*r*) is the law of force field, then by a formula from classical dynamics, we have

$$\frac{d^2u}{a\theta^2} + u = \frac{f(r)}{h^2u^2}$$

with the usual notations

$$\Rightarrow f(r) = \frac{MG}{\left(r - r_1\right)^2}$$

This represents the inverse square law but r is the distance of the body from the centre of mass of the subsystem excluding the test body of mass m. Since $r - r_1 < r$ the distance $r - r_1$ must be the distance from the overall centre of mass of the full system including the test mass m to m Thus, Newton's inverse law and the law of gravitation are always true and the law of force-field is

$$f(r) = \frac{MG}{[r]^2} \tag{8}$$

Where $[r]$ is the radial distance of the test mass m from the common centre of mass ⟹ the potential function is

$$\varnothing = \frac{MG}{[r]} \tag{9}$$

Orbit of Planetary Motion

For $r_1 \neq 0$ we proceed as follows. Expanding the RHS of (7) by using the binomial series

$$\therefore \frac{d^2u}{a\theta^2} + u = \frac{MG}{h^2}(1 + 2r_1, u + 3r_1^2u^2 + \ldots$$

$$i.e. \frac{d^2u}{a\theta^2} + \left(1 - 2MGr_1h^{-2}\right)u = MGh^{-2}\left(1 + 3r_1^2u^2 + \ldots\right)$$

By letting

$$\phi = \left(-\frac{MGr}{} \right)$$

the last equation becomes

$$\frac{d^2u}{d\phi^2} + u = \frac{MG}{h^2}\left(1 - \frac{2MGr_1}{h^2}\right)^{-1}(1 + 3r_1^2u^2 + \cdots)$$

Discarding higher powers containing u^3, u^4 etc. the above equation becomes

$$\frac{d^2u}{d\phi^2} + u = \frac{MG}{h^2} + \frac{\mu_1 M}{4\pi}u^2 \tag{10}$$

which is the same as the equation for planetary motion obtainable from Newton's law [5]

$$\frac{d\overline{P}}{dt} = m\overline{E}_1$$

where $\overline{E}_1 = -\dfrac{MI}{4\pi\epsilon_1 R^2}$ by using $m = m_0 \text{Exp}\left(\dfrac{v^2}{2c^2}\right)$

This shows that the modified Newton's theory gives non-ellipse orbit as in GRT, but the latter doesn't consider the centre of mass, retarded potentials, the Lorentz Force equation etc. Further GTR has the disadvantage that M.W. Evans has refuted the Einstenian GRT [7].

Derivation of Momentum Velocity Relation

$p^2 = m^2v^2 + h^2\overline{k}^2$ and Mass-Energy Relation:

$E^2 = m^2c^4 + h^2w^2$ where $m = m_0 \text{Exp}\left(\dfrac{v^2}{2c^2}\right)$

In the Compton-effect experiment, it was observed that, when a photon of energy $\hbar w = p_0 c$, strikes an electron of rest mass m_0, both will be deflected, and if the former makes angle θ and the latter an angle φ with the initial direction of photon, then by law of conservation of energy [8].

$$(m - m_0)c^2 = \hbar(\omega - \omega') = (p_0 - p_1)c \tag{11}$$

where p_0, p_1 are the initial and final momentum of the photon and m is the mass of the electron in motion.

$$\therefore p_0 - p_1 = (m - m_0)c \tag{12}$$

Also, by the law of conservation of momentum, if $\boldsymbol{p}$ is the momentum of the electron, then

$$p_1 \cos\theta + p\cos\varphi = p_0$$

$$p_1 \sin\theta = p\sin\varphi$$

$$\therefore \boldsymbol{p}^2 = (p_0 - p_1)^2 + 2p_0p_1(1 - \cos\theta) \tag{13}$$

$$\text{But } m^2c^2 - m^2\boldsymbol{v}^2 = (c^2 - v^2)m_0^2 \exp\left(\frac{\boldsymbol{v}^2}{c^2}\right)$$

$$\approx m_0^2c^2\left(1 - \frac{\boldsymbol{v}^2}{c^2}\right)\left[1 + \frac{\boldsymbol{v}^2}{c^2} + \frac{\boldsymbol{v}^4}{2c^4}\right]$$

$$= m_0^2c^2\left(1 - \frac{\boldsymbol{v}^4}{2c^4} - \frac{\boldsymbol{v}^6}{3c^6}\right)$$

$$i.e.\ m^2c^2 - m^2v^2 \approx m_0^2c^2\left(1 - \frac{\boldsymbol{v}^4}{2c^4}\right) \tag{14}$$

From (12) $mc = m_0c + (p_0 - p_1)$

$$\therefore m^2c^2 - \boldsymbol{p}^2 = \left[m_0c + (p_0 - p_1)\right]^2 - (p_0 - p_1)^2 - 2p_0p_1(1 - \cos\theta)$$

$$= m_0^2c^2 + 2m_0(p_0 - p_1)c - 2p_0p_1(1 - \cos\theta)$$

$$= m_0^2c^2 + 2m_0(m - m_0)c^2 - 2p_0p_1(1 - \cos\theta) \text{ by } (12)$$

$$\text{But } m - m_0 = m\left[1 - \exp\left(-\boldsymbol{v}^2/2c^2\right)\right] \approx \left(m\boldsymbol{v}^2/2c^2\right)$$

$$\therefore m^2c^2 - \boldsymbol{p}^2 = m_0^2c^2 + 2m_0c^2\left(m\boldsymbol{v}^2/2c^2\right) - 2p_0p_1(1 - \cos\theta)$$

$$i.e.\ m^2c^2 - \boldsymbol{p}^2 = m_0^2c^2 + m_0m\boldsymbol{v}^2 - 2p_0p_1(1 - \cos\theta) \tag{15}$$

As a first approximation we may take $\boldsymbol{p} \approx m\boldsymbol{v}$ ∴ LHS of equations (14) and (15) are equal.

∴ RHS must be approximately equal.

$$\therefore m_0^2c^2\left(1 - \frac{\boldsymbol{v}^4}{2c^4}\right) \approx m_0^2c^2 + m_0m\boldsymbol{v}^2 - 2p_0p_1(1 - \cos\theta)$$

$$\therefore 2\, p_0\, p_1 (1-\cos\theta) \approx m_0 m \boldsymbol{v}^2 + \frac{1}{2} m_0^2\, v^4 c^{-2}$$

$$\approx m_0 \boldsymbol{v}^2 \left[m + \left(\frac{m_0 \boldsymbol{v}^2}{2c^2} \right) \right]$$

$$= m_0^2\ \boldsymbol{v}^2 \left(\exp \cdot \frac{\boldsymbol{v}^2}{2c^2} + \frac{\boldsymbol{v}^2}{2c^2} \right)$$

$$\approx m_0^1\, \boldsymbol{v}^2 \left(1 + \frac{\boldsymbol{v}^2}{2c^2} + \frac{\boldsymbol{v}^2}{2c^2} \right)$$

$$= m_0^2\, \boldsymbol{v}^2 \left(1 + \frac{\boldsymbol{v}^2}{c^2} \right)$$

$$\approx m_0^2\, \boldsymbol{v}^2 \exp \left(\frac{\boldsymbol{v}^2}{c^2} \right)$$

$$= m^2\, \boldsymbol{v}^2 \qquad (16)$$

using (16) in (13) the latter equation becomes

$$\boldsymbol{p}^2 = (p_0 - p_1)^2 + m^2\, \boldsymbol{v}^2$$

which is of the form

$$\boldsymbol{p}^2 = m^2\, \boldsymbol{v}^2 + \hbar^2 \boldsymbol{k}^2 \qquad (17)$$

where $|\hbar \bar{\boldsymbol{k}}| = p_0 - p_1$ i.e. momentum consists of a mechanical part mv and a quantum-mechanical part/electromagnetic part $i\hbar \boldsymbol{k}$; so, we can write

$$\boldsymbol{p}^* = m\boldsymbol{v} \pm i\hbar \boldsymbol{k} = m\boldsymbol{v} \pm iq\mathrm{A}$$

$$= m\left(\boldsymbol{v} \pm \frac{iq}{m} \mathbf{A} \right) \qquad (18a)$$

$$= m\left(\boldsymbol{v} \pm i\boldsymbol{v}_p \right) \qquad (18b)$$

Equation (14) can be rewritten as

$$\left[m^2\, c^2 + \hbar^2\, \boldsymbol{k}^2 + \frac{1}{2} m_0^2 \frac{v^4}{c^2} \right] - \left(m^2\, \boldsymbol{v}^2 + \hbar^2 \boldsymbol{k}^2 \right) = m_0^2\, c^2$$

which is of the form $\mathrm{E}^2 c^{-2} - \boldsymbol{p}^2 = m_0^2\, c^2$ or $\mathrm{E}^2 - \boldsymbol{p}^2 c^2 = m_0^2\, c^4$

where $\mathrm{E}^2 = m^2 c^4 + \hbar^2 \omega^2$ and $\hbar^2 \omega^2 = \hbar^2 \boldsymbol{k}^2 c^2 + \frac{1}{2} m_0^2 v^4$

$\therefore$ Energy consists of an inertial part mc^2 and a quantum-mechanical part/electromagnetic part $i\hbar\omega$. It is thus possible to take

$\bar{\mathrm{E}} = mc^2 - i\hbar\omega = mc^2 - i\hbar(\partial / \partial t)\log\, \psi)$

$$i.e. \bar{\mathrm{E}}\psi = mc^2 \psi - i\hbar \left(\frac{\partial}{\partial t} \right) \psi \qquad (19)$$

and $\bar{\boldsymbol{p}} = m\boldsymbol{v} + i\hbar\boldsymbol{k} = m\boldsymbol{v} + i\hbar \left(\frac{\partial}{\partial \boldsymbol{r}} \right) (\log \psi)$

$$i.e.\ \bar{\boldsymbol{p}}\psi = m\boldsymbol{v}\psi + i\hbar \left(\frac{\partial}{\partial \boldsymbol{r}} \right) \psi \qquad (20)$$

Application of Lorentz Force Equation

The Lorentz Force law is [1]

$$-\frac{d\bar{P}}{dt} = m\bar{E}_1 + m\bar{v} \times \bar{B}_1$$

for a mass particle and hence

$$\frac{d\bar{P}}{dt} = q\bar{E}_2 + m\bar{v} \times \bar{B}_2$$

for motion of an electron.

For an electron (having both mass and charge), we use (18)(a).

Hence, we replace $\boldsymbol{v}$ by $\boldsymbol{v} \pm i\frac{q}{m}\mathbf{A}$ for the electron and $\boldsymbol{v} \pm i\boldsymbol{v}_p$ for the mass particle. Equivalently we replace $\boldsymbol{p}$ by $m\boldsymbol{v} \pm iq\mathbf{A}$ for electron and $\boldsymbol{p}$ by $m\boldsymbol{v} \pm im\boldsymbol{v}_p$ for a mass particle in the above law. Dropping the subscripts 1 and 2, we have

$\frac{d}{dt}(m\boldsymbol{v} \pm iq\mathbf{A}) = q\mathbf{E} + q(\boldsymbol{v} \pm i\boldsymbol{v}_p) \times \mathbf{B}$ for electron and 21 (a)

$\frac{d}{dt}(m^{-}\boldsymbol{v} \pm im\boldsymbol{v}_p) = m\mathbf{E} + m(\boldsymbol{v} \pm i\boldsymbol{v}_p) \times \mathbf{B}$ for mass particle 21 (b)

Equating real parts $\frac{-d}{dt}(m\boldsymbol{v}) = m\mathbf{E} + m\boldsymbol{v} \times \mathbf{B}$ for mass particle

$\frac{d}{dt}(m\boldsymbol{v}) = q\mathbf{E} + q\boldsymbol{v} \times \mathbf{B}$ for electron

$i.e. -\frac{d\boldsymbol{v}}{dt} \approx \mathbf{E} + \boldsymbol{v} \times \mathbf{B}$ for mass particle 22 (a)

and

$\frac{d\boldsymbol{v}}{dt} \approx \frac{q}{m}(\mathbf{E} + \boldsymbol{v} \times \mathbf{B})$ for electron 22 (b)

by assuming $\frac{dm}{dt} \approx 0$

Equating imaginary parts of (21), we have

$q\frac{d\mathbf{A}}{dt} = q\boldsymbol{v}_p \times \mathbf{B}$ $i.e. \frac{d\mathbf{A}}{dt} = \boldsymbol{v}_p \times \mathbf{B}$ for electron 23 (a)

and $\frac{d}{dt}(m\boldsymbol{v}_p) = -m\boldsymbol{v}_p \times \mathbf{B}$ $i.e.\ \frac{d\boldsymbol{v}_p}{dt} \approx -\boldsymbol{v}_p \times \mathbf{B}$ for mass particle 23 (b)

Now equation (23)(a) for the electron becomes, by using polar co-ordinates*

$$\frac{d\mathbf{A}}{dt} = \left(\dot{r}\mathbf{I} + r\dot{\theta}\mathrm{J} \right) \times \mathrm{B}(\mathbf{I} \times \mathrm{J})$$

$\frac{d}{dt}\left(\frac{mv_p}{q}\right) = (\dot{r}\mathbf{I} + r\dot{\theta}\mathbf{J}) \times \mathrm{B}\mathbf{K}$, Since $m\boldsymbol{v}_p = q\mathbf{A}$

$$\frac{d}{dt}\left(m\boldsymbol{v}_p\right) = q\mathrm{B}\left(-\dot{r}\mathrm{J} + r\dot{\theta}\mathbf{I}\right)$$

$$\frac{d}{dt}\left(\dot{r}\mathbf{I} + r\dot{\theta}\mathrm{J}\right) = \frac{-q\mathrm{B}}{m}\dot{r}\mathrm{J} + \frac{q\mathrm{B}}{m}r\dot{\theta}\mathbf{I}$$

$$\therefore \frac{d\dot{r}}{dt} = \frac{q\mathrm{B}}{m}r\dot{\theta} \tag{24}$$

$$\text{and } \frac{d}{dt}\left(r\dot{\theta}\right) = \frac{-q\mathrm{B}}{m}\dot{r} \tag{25}$$

*Here we have changed the usual formula for the radial and transverse components of *acceleration of a particle* by the following considerations. The apparent position of fast revolving leaves of a fan can be 'everywhere' within (0,2π). Similarly, an electron in the shape of a spherical shell moving about a mean position has the appearance of a cloud having both radial and transverse velocities and hence the formulae for acceleration of a particle are no longer applicable hence are replaced by $\frac{d\dot{r}}{dt}$ and $\frac{d}{dt}\left(r\dot{\theta}\right)$. In other words, $\boldsymbol{v}_p$ is not localized but existing simultaneously at all points on a sphere. That is, $\frac{d\mathbf{I}}{dt} = 0$ and $\frac{d\mathbf{J}}{dt} = 0$, for electron motion.

Thus, we get equations (24) and (25). On integration, equation (25) gives

$$r\dot{\theta} = \frac{-q\mathrm{B}\,r}{m} + \mathrm{C}$$

$$= -\omega r + \mathrm{C} \text{ where } \frac{q\mathrm{B}}{m} = \omega$$

$$\therefore \frac{q\mathrm{B}}{m}r\dot{\theta} = \omega\left(-\omega r + \mathrm{C}\right)$$

$$\therefore \text{Equation}\,(24) \text{ becomes } \ddot{r} = \omega\,(-\omega r + c)$$

i.e. $\left(\mathrm{D}^2 + \omega^2\right)r = \omega\mathrm{C}$. Solving this differential equation, we get

$$r = \lambda\cos\left(\omega t + \varepsilon\right) + \frac{\mathrm{C}}{\omega} = \mathrm{R}_0 + \lambda\cos\left(\omega t + \varepsilon\right) \tag{26}$$

where λ, ε, R_0 are constants of integration

$$\dot{r} = -\lambda\omega\sin\left(\omega t + \varepsilon\right) \tag{27}$$

$$\therefore r\dot{\theta} = -\lambda\omega\cos\left(\omega t + \varepsilon\right) \tag{28}$$

Equations (26) to (28) indicate a simple harmonic motion about a mean position R_0. The solution of equation (22)(a) is the same as the equation for planetary motion. Thus, the solution of equation (21)(a) consists of superposition of an elliptic motion and a time dependent simple harmonic motion.

A Quantum Mechanical Partitioning of Space around a Mass/Charge

We shall take $\mathrm{R}_0 = \lambda\left(n+1\right)$ and $\lambda = r_1 n$, where r_1 is Bohr radius

$$\therefore \mathrm{R}_0 = n\left(n+1\right)r_1 \tag{29}$$

or equivalently we may write $r_n = n\left(n+1\right)r_1$, in order to have similarity with Bohr's principle. This partitioning of space will be used in the next section. (In the theory of hydrogen atom Bohr took $r_n = n^2 r_1$). From equations (27) and (28) we have

$$v_p = \sqrt{\dot{r}^2 + r^2\dot{\theta}^2} = \lambda\,\omega$$

$\therefore m v_p = m\,\lambda\,\omega$ and $\lambda\left(m\boldsymbol{v}_p\right) = m\,\lambda^2\,\omega = \hbar$ (where ħ is Planck constant) in accordance with De Broglie hypothesis [8]. Hence, ħ can be interpreted as quantum-mechanical angular momentum, $\boldsymbol{v}_p$ as phase velocity and λ as wavelength of electron cloud.

Resolution of Singularity of Field Energy

In this section, we attempt to remove the singularity of field energy due to an electron/mass particle, by using Bohr's principles. In electromagnetism, we have the equation that the electromagnetic field energy of a point charge of radius a is given by $\mathrm{U}_{elec} = \frac{q^2}{8\pi\varepsilon_0 a}$ [8]. Similarly, the energy of a charged sphere of radius a is also given by $\mathrm{U}_{elec} = \frac{\mathrm{Q}^2}{8\pi\varepsilon_0 a}$. Therefore, total energy in the field and the energy within the charge are each equal to $\frac{\mathrm{Q}^2}{8\pi\,\varepsilon\mathrm{A}}$. When a tends to zero, the conclusion is that there is an infinite amount of energy in the field of a point charge or charged sphere. Applying Bohr's theory of hydrogen atom, we can remove the apparent singularity stated above. We can re-write the above formula in the form

$$\mathrm{U}_{elec} = \frac{\mathrm{Q}^2}{8\pi\varepsilon_0 a} = \frac{\mathrm{Q}^2}{8\pi\varepsilon_0}\sum_1^{\infty}\frac{1}{n\left(n+1\right)a} \text{ since } \sum_1^{\infty}\frac{1}{n\left(n+1\right)} = 1$$

$$= \sum_1^{\infty}\frac{\mathrm{Q}^2}{8\pi\varepsilon_0 r_n} = \sum_1^{\infty}\mathrm{E}_n$$

where $\mathrm{E}_n = \frac{\mathrm{Q}^2}{8\pi\varepsilon_2 r_n}$, and ε_2 is our notation for ε_0 from EM theory, E_n is the energy level at a radial distance $r_n = n\left(n+1\right)a$, $a = r_0$ or an integral multiple of r_0 or an integral multiple of r_0 and r_0 is the Bohr radius. But Bohr uses $r_n = n^2 r_0$ instead of $n\left(n+1\right)r_0$. Thus, the field energy at r_n from a charge is E_n and the total energy in the field is $\mathrm{U} = \frac{\mathrm{Q}^2}{8\pi\varepsilon_2 a}$. We can extend this result to mass particle of radius a. The field energy due to a mass particle of mass M may be taken $\mathrm{U} = \frac{\mathrm{M}^2}{8\pi\varepsilon_1 a}$. When $a \to 0$, we replace M by [M] and Q by $\left(\frac{\mathrm{Q}}{\mathrm{M}}\right)[\mathrm{M}]$, so that $\mathrm{U} \to 0$ as $a \to 0$.

Interpretation of Schrodinger's Wave Function

From the theory of Maxwell-Lawrentz' equations for a mass particle/electron we have noted that the four potentials is a measure of the energy momentum felt by a charge q in the $q(\phi_2, \mathbf{A}_2)$ four-field $(\phi_2, \mathbf{A}_2)$ of a source charge Q [1]. Similarly, $m(\phi_1, \mathbf{A}_1)$ is a measure of the energy-momentum felt by a mass-particle m in the four-field $(\phi_1, \mathbf{A}_1)$ of mass M. From equation (18) and equation (19) we can infer that the total momentum and energy of a charged particle consists of a (i) Mechanical/Inertial Part (ii) Quantum Mechanical Part/Electromagnetic Part.

For an electron in an atom, mc^2 and $q\phi_2$ represent a measure of the energy due to (i) Inertial Mass m and (ii) Electrostatic potential of the nucleus. Let ρ denote mass density so that ρc^2 is energy density; therefore, energy momentum density due to mass content of the moving electron in the orbit is $(\rho c^2, \rho \boldsymbol{v})$. If ρ_c is charge density of the electron then the energy momentum density due to charge of the moving electron is $\rho_c(\phi_2, \mathbf{A}_2) = (\rho_c \phi_2, \rho_c \mathbf{A}_2)$, with the notations of Maxwell-Lorentz' theory. By the equation of continuity and the gauge condition

$$\frac{1}{c^2}\frac{\partial}{\partial t}\left(\rho c^2\right) + \operatorname{div}(\rho \boldsymbol{v}) = 0 \qquad (30)$$

$$\frac{1}{c^2}\frac{\partial}{\partial t}\left(\rho_c \phi_2\right) + \operatorname{div}(\rho_c \mathbf{A}_2) = 0 \qquad (31)$$

Combining these equations

$$\frac{\partial}{\partial t}\left(\rho + ic^{-2}\rho_c\phi_2\right) + \operatorname{div}\left(\rho\boldsymbol{v} + i\rho_c\mathbf{A}_2\right) = 0 \qquad (32)\ (a)$$

$$i.e.\ \frac{\partial \rho^*}{\partial t} + \operatorname{div}\boldsymbol{p}^* = 0 \qquad (32)\ (b)$$

$$\text{where } \boldsymbol{p}^* = \rho\boldsymbol{v} + i\rho_c\mathbf{A}_2, \qquad (33)\ (a)$$

$$\text{and } \rho^* = \rho + ic^{-2}\rho_c\phi_2 = \rho_{mech} + i\rho_{qm} \qquad (33)\ (b)$$

are all densities.

By Newton's law of motion,

$$\frac{d\boldsymbol{p}^*}{dt} = -\nabla\left(\rho^* c^2\right) = -\nabla\left(\rho c^2 + i\rho_c\phi_2\right) \qquad (34)$$

Taking divergence, $\operatorname{div}\dot{\boldsymbol{p}}^* = -c^2\nabla^2\left(\rho^*\right)$ (35)

By defining L* and H* as Lagrangian and Hamiltonian densities,

we have $\mathrm{L}^* = \boldsymbol{p}^*.\boldsymbol{v} - \mathrm{H}^*, d\mathrm{S}^* = \mathrm{L}^* dt = \boldsymbol{p}^*.d\boldsymbol{r} - \mathrm{H}^* dt$

$$\therefore \nabla \mathrm{S}^* = \boldsymbol{p}^* \text{and} - \frac{\partial \mathrm{S}^*}{\partial t} = \mathrm{H}^* = \rho^* c^2$$

$$\therefore \nabla^2 \mathrm{S}^* = \operatorname{div}\boldsymbol{p}^* = -\frac{\partial \rho^*}{\partial t} = -\frac{\partial}{\partial t}\left(-\frac{1}{c^2}\right)\left(\frac{\partial \mathrm{S}^*}{\partial t}\right) = \frac{1}{c^2}\frac{\partial^2 \mathrm{S}^*}{\partial t^2}$$

$$\therefore \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathrm{S}^* = 0$$

From (32)(b), by differentiating partially w.r.t. t

$$\frac{\partial^2 \rho^*}{\partial t^2} + \operatorname{div}\left(\frac{\partial \boldsymbol{p}^*}{\partial t}\right) = 0 \qquad (36)$$

$$i.e.\ \frac{\partial^2 \rho^*}{\partial t^2} + \operatorname{div}\left[\left(\frac{d\boldsymbol{p}^*}{dt}\right) - (\boldsymbol{v}.\nabla)\boldsymbol{p}^*\right] = 0$$

$$i.e.\ \frac{\partial^2 \rho^*}{\partial t^2} - c^2\nabla^2\rho^* = \operatorname{div}\left[(\boldsymbol{v}.\nabla)\boldsymbol{p}^*\right]$$

$$i.e.\ \frac{1}{c^2}\frac{\partial^2 \rho^*}{\partial t^2} - \nabla^2\rho^* = c^{-2}\operatorname{div}\left[(\boldsymbol{v}.\nabla)\boldsymbol{p}^*\right] \qquad (37)$$

Again from 32 (b), $\frac{\partial \boldsymbol{p}^*}{\partial t} + (\boldsymbol{v}.\nabla)\boldsymbol{p}^* = -c^2\nabla\rho^*$

$$\therefore \frac{\partial^2 \boldsymbol{p}^*}{\partial t^2} + \frac{\partial}{\partial t}\left[(\boldsymbol{v}.\nabla)\boldsymbol{p}^*\right] = -c^2\nabla\left(\frac{\partial \rho^*}{\partial t}\right) = c^2\nabla\left(\nabla\cdot\boldsymbol{p}^*\right) =$$

$$c^2\left[\nabla^2\boldsymbol{p}^* + \nabla\times\left(\nabla\times\boldsymbol{p}^*\right)\right] \qquad (38)$$

$$i.e.\ \frac{\partial^2 \boldsymbol{p}^*}{\partial t^2} - c^2\nabla^2\boldsymbol{p}^* = -\frac{\partial}{\partial t}\left[(\boldsymbol{v}.\nabla)\boldsymbol{p}^*\right] + c^2\nabla\times\left(\nabla\times\boldsymbol{p}^*\right)$$

$$i.e.\ \frac{1}{c^2}\left(\frac{\partial^2 \boldsymbol{p}^*}{\partial t^2}\right) - \nabla^2\boldsymbol{p}^* = -\frac{1}{c^2}\frac{\partial}{\partial t}\left[(\boldsymbol{v}.\nabla)\boldsymbol{p}^*\right] + \nabla\times\left(\nabla\times\boldsymbol{p}^*\right) \qquad (39)$$

As an approximation, the RHS of (38) and (39) are negligible. Therefore, these equations become the homogenous wave equations.

$$\frac{1}{c^2}\frac{\partial^2\left(\rho^* c^2\right)}{\partial t^2} - \nabla^2\left(\rho^* c^2\right) \approx 0 \qquad (40)$$

Where, $\rho^* = \rho + ic^{-2}\rho_c\phi_2 = \rho_{mech} + i\rho_{qm}$

$$\text{or } \rho^* c^2 = \rho^*_{mech}c^2 + i\rho^*_{qm}c^2$$

$$\text{and } \frac{1}{c^2}\frac{\partial^2 \boldsymbol{p}^*}{\partial t^2} - \nabla^2\boldsymbol{p}^* \approx 0 \qquad (41)$$

Both real and imaginary parts of (40) and (41) satisfy the homogeneous wave equation. To solve (40) by the method of separation of variables,

we substitute

$$\rho^* c^2 = \psi(x, y, z)\mathrm{T}(t) \qquad (42)$$

where $\psi(x,y,z) = \psi_1(x,y,z) + i\psi_2(x,y,z)$, into (40).

Taking imaginary parts, we get

$$\frac{1}{c^2}\psi_2(x,y,z)\mathrm{T}'' = \left(\nabla^2\psi_2\right)\mathrm{T}$$

$$i.e.\ \frac{\nabla^2\psi_2}{\psi_2} = \frac{\mathrm{T}''}{c^2\mathrm{T}} \quad \text{Let each} = -k^2$$

$$\therefore \left(\nabla^2 + k^2\right)\psi_2 = 0 \qquad (43)$$

and $$T'' + k^2c^2T = 0 \qquad (44)$$

The last equation has solutions $T = A\exp(\pm\, ikct)$ and then $|T| = A$

$$\therefore \rho_{qm}c^2 = \psi_2(x,y,z)\,A\exp(\pm\, ikct) \qquad (45)\ (a)$$

and $$\left|\rho_{qm}c^2\right| = A\left|\psi_2(x,y,z)\right| \qquad (45)\ (b)$$

$\therefore \psi_2(x,y,z)$ has the dimension of $\rho_{qm}c^2$

Therefore, the wave function $\psi_2(x,y,z)$ represents quantum mechanical energy density. From classical dynamics, we have T + V= constant = E where T is the kinetic energy and V is potential energy. Take V(r) as the potential energy per unit charge at the electron due to the nucleus of the atom. By using the mass velocity relation $m = m_0 \mathrm{Exp}\left(\frac{v^2}{2c^2}\right)$ and substituting $T = (m - m_0)c^2$, the energy equation gives

$$m_0c^2\mathrm{Exp}\left(\frac{v^2}{2c^2}\right) - m_0c^2 + qV(r) = E \qquad (46)$$

$$\therefore \mathrm{Exp}\left(\frac{v^2}{2c^2}\right) = 1 + \frac{E - qV(r)}{m_0c^2}$$

$$\therefore \boldsymbol{p}^2 = m_0^2\boldsymbol{v}^2\mathrm{Exp}\left(v^2/c^2\right) = m_0^2\cdot 2c^2\log\left[1+\frac{E-qV(r)}{m_0c^2}\right]\left[1+\frac{E-qV(r)}{m_0c^2}\right]^2$$

$$= 2m_0^2c^2\left[\frac{E-qV(r)}{m_0c^2} - \frac{1}{2}\left(\frac{E-qV(r)}{m_0c^2}\right)^2 + \cdots\right]\left[1+\left(\frac{E-qV(r)}{m_0c^2}\right)\right]^2$$

Letting $\boldsymbol{p} = \hbar\boldsymbol{k}$ implies

$$\hbar^2\boldsymbol{k}^2 = 2m_0\left(E - qV(r)\right)\left[1 - \frac{E-qV(r)}{2m_0c^2} + \cdots\right]\left[1+\frac{E-qV(r)}{m_0c^2}\right]^2$$

$$\therefore \boldsymbol{k}^2 = \frac{2m_0}{\hbar^2}\left[E - qV(r)\right]\left[1 + \frac{3}{2}\left(\frac{E-qV(r)}{m_0c^2}\right) + \cdots\right] \qquad (47)$$

By using this approximation in equation (43) we get

$$\nabla^2\psi_2 + \frac{2m_0}{\hbar^2}\left[E - qV(r)\right]\left[1 + \frac{3}{2}\left(\frac{E-qV(r)}{m_0c^2}\right) + \cdots\right]\psi_2 = 0 \qquad (48)$$

By omitting second and higher powers of, E − qV (r) we get the Schrodinger's equation [2]

$$\nabla^2\psi_2 + \frac{2m_0}{\hbar^2}\left[E - qV(r)\right]\psi_2 = 0 \qquad (49)$$

Thus, the Schrodinger's wave function ψ_2 satisfying (43) and (49) has the dimension of quantum mechanical energy density. Substituting the solution of Schrodinger's equation (49) in to (45) (a), we get the imaginary part of the solution of equation (40). Similarly, we can find the real part of the solution of equation (40).

Conclusions

The motion of a particle in an n -particle system can be handled by using Newtonian theory along with the centre of mass coordinate system but this possibility is ignored in GTR. The concepts of retarded potentials, vector potentials and the Lorentz Force equations can be used in the modified Newtonian theory whereas these concepts are not treated in GTR. Hence GTR cannot be considered as a generalized version of modified Newtonian dynamics. Besides M.W. Evans has definitively refuted Einstenian general relativity [7].

References

1. Chandramohanan MR (2010) On Maxwell-Lawrentz' equations for a mass-particle in motion. Proceedings of Natural Philosophy Alliance, 17th Annual conference at California State University, Long Beach USA pp: 660-664.
2. Merzbacher Eugen (1998) Quantum Mechanics. https://www.wiley.com/en-gb/Quantum+Mechanics%2C+3rd+Edition-p-9780471887027.
3. Whittaker ET (1960) A History of the Theories of Aether and Electricity. https://www.abebooks.com/book-search/title/historytheories-aether-electricity/used/book/.
4. Chandramohanan MR (2024) Field Dependant Metric for Gravitational/EM Fields and a Non-Linear Transformation for Local to Proper Space-Time World. Journal of Physics & Optics Science 6: 1-4.
5. Chandramohanan MR (2024) About some applications of Mathematics. Journal of Physics & Optics Science 6: 1-7.
6. Lanczos Cornelius (1964) The Variational Principles of Mechanics. https://books.google.co.in/books/about/The_Variational_Principles_of_Mechanics.html?id=GRHXzEUkZl8C&redir_esc=y.
7. Evans MW (2015) Definitive Refutations of Einsteinian General Relativity. https://scholar.google.com/citations?view_op=view_citation&hl=en&user=UzKFt2QAAAAJ&citation_for_view=UzKFt2QAAAAJ:x21FZCSn4ZoC.
8. Feynman RP, Leighton RB (1989) Feynman Lectures on physics. https://www.feynmanlectures.caltech.edu/