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Analysis of Mathematical Ethnomodelling Competency Levels of Grade 12 Learners' on Lotka-Volterra Model Predator-Prey Problem

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ABSTRACT

Background: Mathematical ethnomodelling is a prerequisite to conceptualising and comprehending mathematical concepts independently through reflective-critical thinking to solve real-world problems. Learners in grade 12 in their final Further Education and Training (FET) Phase examination in one of the examining bodies have demonstrated knowledge of mathematical modelling competencies in the Lotka-Volterra Model, despite a few challenges.

Aim: To analyse the level of mathematical modelling competency of grade 12 learners, and how they demonstrate knowledge of understanding when solving the Lotka-Volterra model predator-prey problems.

Setting: Data was gathered from 15 grade 12 learners of FET final exams by an examining body in South Africa, these learners were conveniently selected.

Method: The research design is qualitative on document analysis of real-world tasks on the Lotka-Volterra model. The data-gathering instrument of the study is the learners' workings on mathematical modelling tasks.

Findings: The results indicated that 9 learners invoked their modelling competencies in the modelling problem of the Lotka-Volterra model to produce mathematical results. A lack of appropriate mathematical knowledge and techniques may also hinder learners operating at Levels 1 and 2 from moving on to the process of mathematization and mathematical working, even if they could simplify and structure the problem.

Conclusion: We advise teachers and teacher educators to be aware that if the mathematical model is insufficient or if the learners have a poor grasp of mathematics, they might be unable to arrive at a meaningful solution even though they are aware of a strategy that could be used to a task.

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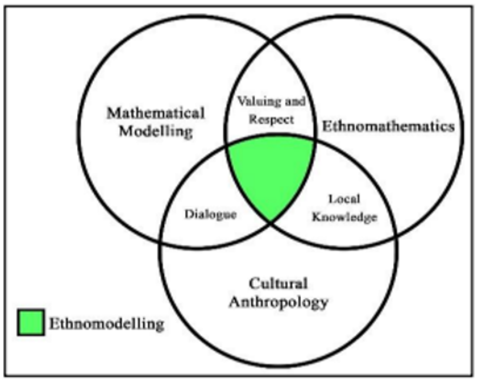
Introduction

The Lotka-Volterra model predator-prey problem is a modelling problem that links mathematics to nature's wildlife. The cultural backgrounds and societal experiences of learners are rarely connected to mathematics teaching in South African schools [1]. It does not draw connections between mathematics and culture based on teaching theory. According to Govender and Machingura mathematical modeling is a problem-solving technique in which real-world contextual situations are represented mathematically through the process of mathematisation [2]. The ability of a learner to create, apply, evaluate, and compare mathematical models to address real-world problems is commonly defined as mathematical modelling competency [3]. The South Africa Department of Basic Education released the country's National Senior Certificate results for 2019 matric learners. These are referred to as the "matric results" and they determine admission and placement into higher institutions [4]. About 81.3% of those who wrote the matriculation exams passed [5]. What was of much concern from these results is what Minister of Education, Angie Motshekga said about the drop-in performance in mathematics. This is one of the most important subjects that is considered critical for the country's economic growth and development [4].

In a pedagogical perspective, ethnomodelling makes it possible to explore ideas and conceptualise mathematics in the classroom as a process of inducting young learners into mathematical concepts of their culture. By inclusivity of a strong representation of customs and issues of a learner's own community, an ethnomodelling perspective has positively reshaped cultural identity through nature. Although, learners to use ethnomodelling in the classroom improve their creativity in mathematics thereby bring back a sense of beautiful engagement with nature's wildlife reserving as their cultural identity. Ethnomodelling is a process of translation and elaboration drawn from reality to mathematics and is best example of an educational approach that respects a diversity of cultural forms of mathematics [6]. Finding mathematical models and incorporating them into mathematics and teaching learners are done without protocols or necessity. Alghar and Radjak posits that to distinguish, integrate and develop cultural mathematical models with formal mathematics in the academic sphere, a new method in ethnomathematics is thus required, hence mathematical ethnomodelling [7]. A thorough examination of mathematical models present in conventional ethnomathematics was required [8]. As a result, ethnomodelling is a novel teaching strategy that incorporates mathematical models of culture was developed [9]. As a useful application of ethnomathematics to mathematical modelling concepts, ethnomodelling is an alternative method that investigates the scope of mathematics developed in a cultural context. Ethnomodellling broadens the experience of academic mathematical concepts and allows for generalised understanding of the mathematical knowledge that members of cultural groups practice Santos & Madruga as cited in [7]. In this context, ethnomodelling does not follow the linear modelling approach that is prevalent in this modern world. Ethnomodelling recognises the need for culturally based views on modelling process [10].

Cultural anthropology, mathematical modelling and ethnomathematics come together to form ethnomodelling [7]. The intersection that ethnomodelling creates necessitates the introduction of mathematical models of culture into classroom to help learners develop an appreciation of the culture they have learned. These models should be studied alongside academic mathematical models especially Lotka-Volterra models.



Scope of Ethnomodelling adopted [7].

Several studies related to ethnomodelling have been conducted, including exploration of mathematical models in cultural ornaments [7]. Ethnomodelling research is also conducted theoretical such as the importance of dialogical approach, political activities that affect ethnomodelling and in-depth discussions on the intersection of ethnomodelling and ethnomathematics [11]. Some researchers have also involved mathematics learning with ethnomodels such as modelling function concepts from the context of nature of predator-prey model [12].

Although, there have been various studies related to ethnomodelling, the quantity of ethnomodelling research has not been massive as mathematical modelling and ethnomathematics. The contribution of ethnomodelling can direct learners' understanding of how mathematical models develop in people's lives and are applied in cultural group. In addition, learning mathematics with ethnomodelling will construct learners' understanding of formal mathematics directly and foster mutual respect for community traditions and local customs [7].

Predator-Prey Model/Formulae

Models of predator-prey are perhaps the fundamental components of ecosystems that have developed from their resources. In order to survive and continue to fight for their existence, species must compete, change, and disperse. The Lotka-Volterra model comprises two differential equations that depict the interactions between predator and prey species in a shared environment. Similarly, the predator-prey model of a basic ecosystem utilizes a non-linear system of differential equations in the field of mathematical ecology.

Mathematicians Vito Volterra of Italy and Alfred J. Lotka of the United States each independently proposed the model [13]. The model can be developed in various formulas for application, depending on the level and educational context. However, the model first makes a number of simplifying assumptions, which include the following according to [14]:

- When there is no predator, the population of prey increases exponentially.
- The size of the prey and predator populations directly relates to the rate of prey death.
- The population of predators dies from starvation when there is no prey.
- There is no complexity in the environment;
- The predator can consume an endless amount of prey.

Evolutionary interactions between predators and prey are possible. A species' ability to reproduce and interact with other species determine how its population changes. Zebra population size, for example, is a time-dependent function of plants and is represented by x. Given that there is an infinite supply of food for this species, it makes sense that the rate of population growth would increase proportionately to the size of the current population because there are more possible pairings [15].

We modelled the above information as:

$$\frac{dx}{dt} = \beta_1 x \Longrightarrow x(t) = Ae^{\beta_1 t}$$

where β_1= the growth rate of prey population

A=x(0), the initial population size of prey

$\Longrightarrow \frac{dx}{dt} =$ growth rate of prey population with respect to time.

This model's obvious flaw is that, unlike in reality, population growth in it is unrestricted over time. To fix this issue, one could say that as x increases, the growth rate β_1 becomes a function of the population size decreasing. As an alternative, we may model a second population, y(t), that corresponds to a different species, such as lions, who hunt zebras. In this instance, the number of Lions y multiplied by the number of Zebras x will cause the population of Zebras, x, to decline proportionately.

That is, the quantity of interactions between the two species might result in a melancholy little zebra's funeral. The format of this law will be:

$$\frac{dx}{dt} = \beta_1 - C_1 xy$$

Where C_1 is the frequency of fatal interactions. Therefore, it is also necessary to model the evolving predator population, y(t), in this scenario. As its food supply becomes scarce, we predict that it will decline in the absence of prey. So, the equation becomes:

$\frac{dx}{dt} =$ $-\alpha_2$ y where α_2 is the predator mortality rate.

$\frac{dx}{dt} =$ growth rate of predator population with respect to time

Predator birth rate, however, opposes the decline in predator population in the presence of prey. The model becomes: The predator birth rate is determined by multiplying the number of predators (y = lions) by the number of prey (x = zebras) and by the predator's ability to reproduce. The model becomes:

$$\frac{dy}{dt} = -\alpha_2 y + C_2 xy$$

where C_2 is the predator's growth rate or ability to produce offspring from food. These presumptions lead to the representation of the population change over time in a set of two ordinary differential equations, like these:

$$\frac{dx}{dt} = \beta_1 - C_1 xy \text{ and } \frac{dy}{dt} = -\alpha_2 y + C_2 xy$$

The Lotka-Volterra model (Predatory-Prey) system is the name given to this set of equations.

The context in which a model is used and the issues it attempts to solve determine its usefulness. Proofs of theorems are one example of how mathematics has the advantage of certainty. Mathematicians will find this very useful as it reduces the amount of time they have to spend debating the facts. Everyone has an obligation to accept a mathematical claim once it has been proven. Nevertheless, this certainty is exclusive to mathematical assertions; mathematical modelling is not covered by it. Proven results are true—but only for the model. Mathematical proof can be used to validate results about a model. In so far as the model's behavior resembles that of the real world, conclusions derived from it are valid for that context [16].

Theoretical Framework

The application of Lotka-Volterra formulae in the educational setting will help students if they follow the pedagogical mathematical modelling process framework represented in Table 1 consisting of a number of discrete moves that a learner might follow to realize a meaningful solution to a practical problem.

Table 1: Discrete Levels in Pedagogical Mathematical Modelling Process Framework [17].

Categories of Pedagogical Activity	Description
Reading (1)	Understanding and unpacking the information.
Modelling (2)	Changing the task's context from the real world to the mathematical model (Mathematisation)
Estimating (3)	Making meaning of the problem's quantitative estimations in the context.
Calculating (4)	Calculating the missing data on the drawn diagram/or given models using simple mathematical ideas.
Reflecting (5)	Identifying mathematics concepts, facts, formulas and theorists that are relevant to the task solution.
Validating (6)	Interpreting, verifying and validating the results, calculations, and models in a real-world setting.
Writing (7)	Providing a succinct explanation of a report's findings and how they relate to the original task and the methods that led to the task's solution.

This pedagogical mathematical framework encompasses the broad phases of modelling cycle. It does not provide a sufficient framework for thorough assessments of the cognitive processes of learners during modelling process [18]. The Pedagogical modelling framework's ability to analyse the mathematical thinking skills of learners participating in a modelling task is limited [19]. Borromeo-Ferri used the term "mathematical thinking skills," which refers to how individual learners use their mathematical abilities to solve the modelling challenge in ways that are unique to them. Table 1 lists the Pedagogical Mathematical Modelling Process framework categories that will be used in this study to assess the mathematical modelling competencies of grade 12 learners on predator-prey problems. Reading, modelling, estimating, calculating, reflecting, validating, and writing are the seven categories provided in the order in which they are operationalised. This framework helps to assess the mathematical modelling moves of grade 12 learners in modelling as they apply rules with good communication (Dialogue) to showcase their ideas from their local knowledge whilst using critical reflective thinking. They will finally value their findings and respect the need for mathematical ethnomodelling in their curriculum. The development of research related to ethnomodelling has brought a positive new colour to mathematics education [20].

Research Design and Methodology

This research is a qualitative document analysis design. Grade 12 learners were conveniently selected for a task-based activity on their FET final examination paper on module 3: Finance and Modelling in South Africa.

In this study, we articulate aspects from the presented mathematical modelling framework and data obtained from an empirical study. To this end, we point out the contributions grade 12 learners engagement with the modelling problem or process of Lotka-Volterra model and shed some light on the issue of focus:

What level of mathematical modelling competency do grade 12 learners demonstrate when solving Lotka-Volterra model problems?

To address the research question adequately, this study adopted a qualitative approach located within the interpretivist paradigm. This qualitative approach allows researchers through fieldnotes and written responses to items to observe, interpret, or make sense of learners' engagement or responses toward phenomena in a given natural setting like a typical mathematical classroom. In this study, the phenomena of solving a problem using the modelling approach have been explored. Using the interpretive paradigm, one could attempt to describe, analyse and interpret features of the phenomena preserving its complexity and communicating the learners' actions [21].

The interpretivist approach is suitable for qualitative empirical study since it enables one to analyse and make sense of learners' process of working solutions, written and verbal actions in great depth through the lens of a selected analytical framework [21].

Creswell asserts that content/document analysis requires the data to be examined and interpreted to elicit meaning, gain understanding and develop empirical knowledge. Leedy and Omrad describe qualitative analysis as the process of working with collected data through the lens of an analytical framework and entails organizing the data into manageable units that could help in synthesizing patterns that will enable one to discover what is important that could be shared with academic community [22]. Thus, qualitative data analysis requires some creativity and innovation on the part of the researcher to enable the development of logical and meaningful categories of data which could be interpreted to produce results not only assist in answering the research question(s) but also that it can be communicated meaningful ways to other interested persons [2].

Research Item

The problem was chosen based on the technical level of mathematics required to solve the problem and the model by the Grade 12 final examination at their Further Education and Training of an examining body in South Africa. Item 5.1 was on module 3 in addition to other modules which were given to them to answer. These particular tasks were on mathematical modelling focusing on the application of proofs in modelling with predator-prey. The problem was already modelled using Lotka-Volterra recursive formulae. Learners are to explain an important term in the two models. They are to also show a proof of a given relation in the model. A phase plot of the predator-prey model was given to determine the increase in parameters of the predator and prey (see Appendix 1 and 2)

Task 5: Suggested Solution

a) The term $bZ_n L_n$ represents the reduction in the Zebra population caused by attacks by Lions.

b) For equilibrium of Lions

$$L_E = L_E + f.bZ_E L_E - CL_E$$

$$\Longrightarrow 0 = L_E\,(f.b.Z_E - C)$$

$$\Longrightarrow 0 = f.b.Z_E - c$$

$$\Longrightarrow \frac{C}{f.b} = \frac{f.b.Z_E}{f.b}$$

$$\Longrightarrow Z_E = \frac{C}{f.b} \;\ldots\ldots(1)$$

For equilibrium of Zebra

$$\Longrightarrow Z_E = Z_E + aZ_E(1 - \frac{Z_E}{K}) - b.Z_E.L_E$$

$$\Longrightarrow 0 = aZ_E(1 - \frac{Z_E}{K}) - b.Z_E.L_E$$

$$\Longrightarrow 0 = Z_E[a(1 - \frac{Z_E}{K}) - b.L_E]$$

$$\Longrightarrow 0 = a(1 - \frac{Z_E}{K}) - b.L_E$$

$$\Longrightarrow b.L_E = a(1 - \frac{Z_E}{K})$$ substituting from

equation (1), we get,

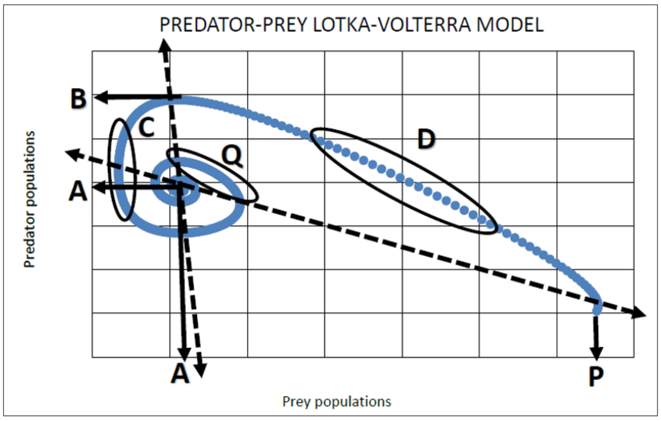
$$\Longrightarrow L_E = \frac{a}{b}(1 - \frac{C}{f.b.K}) \text{ proved}$$

$$\text{C) } 8 = \frac{0{,}8}{0.05}(1 - \frac{1000}{k})$$

$$\Longrightarrow 0.5 = 1 - \frac{1000}{k}$$

$$\Longrightarrow k = 2000$$

Task 4: Suggested Solution



2.1 a) A on phase plot b) B on phase plot

2.2 a) C on phase plot b) D on phase plot

2.3 Pairs of Axes

Accuracy (passing through equilibrium point).
Accuracy (passing through max/min values of prey).
Accuracy (passing through max/min values of predator)

(See Appendix 2)

Analytical Framework used to indicate Moving Levels of modelling competency.

By considering the abilities and skills that characterize each of the moves in the modelling process, the following analytical model which was proposed by Ludiwug and Xu has been adapted and configured for this study [23].

- **Level 0:** the student has not understood the situation and is not able to model the formular or write anything meaningful about the task.
- **Level 1:** the student only understands the given real situation, but is not able to structure and simplify the models or cannot find connections to any mathematical ideas.
- **Level 2:** after investigating the given real situation, the student finds a real model and changes or simplifies the model, but does not know how to transfer this to a mathematical model to suit the real model. The mathematization is partially or completely flawed.
- **Level 3:** the student is able to find not only the real model, but also translate it into a proper mathematical problem, but cannot work with it clearly in the mathematical world.
- **Level 4:** the student is able to pick up a mathematical model from the given model, and work with this model mathematically to produce mathematical results.
- **Level 5:** the student is able to reflect well on the mathematical model from the real situation, work with appropriate mathematical model concepts, and produce and interpret mathematical results in terms of parameters of the given model from which the task emerged.
- **Level 6:** the student is able to experience the mathematical modelling process and validate the solution of the mathematical

task in relation to the given model.

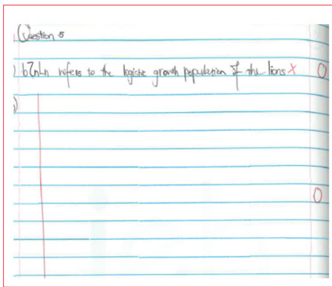
- **Level 7:** student is able to provide a succinct explanation of mathematical concepts and models used to give a better interpretation of the mathematical results.

Results and Discussion

Most of the learners' responses were found to belong to levels 0, 1, 2, or 5. The results and discussions will focus on all levels based on each learner's responses.

Level 0 (Task 5)

Three learners did not write anything concrete or model any related formulae as illustrated by 2 such responses in Figure 1 and Figure 2. They could not read to understand or unpark any meaningful mathematical ideas to tackle the task. The second learner on Figure 2 seemed not to have fully understood or comprehended the given information fully, as to how to substitute the model with the necessary variables before simplifying but instead used numbers with variables to simplify, this is a complete deviation from the problem.



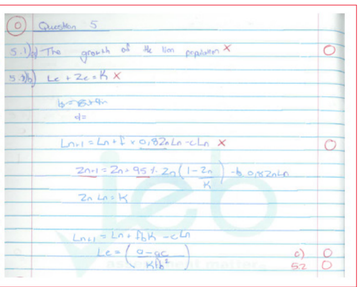


Figure 1: Learner 1 Response **Figure 2:** Learner 2 Response

Level 1(Task 5)

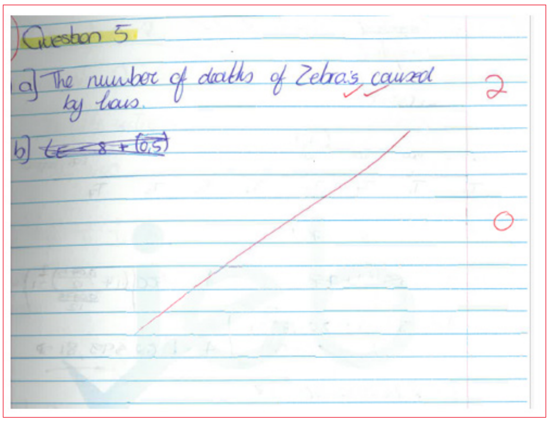


Figure 3: Learner 3 Response

The analysis of learner responses showed that three learners were placed in the Level 1 category. The learners only understand the given real situation but are not able to structure and simplify the models or cannot find connections to any mathematical ideas. They could only give the meaning of the term $bZ_n L_n$ represents the reduction in the Zebra population caused by attacks by Lions.

Level 2 (Task 5)

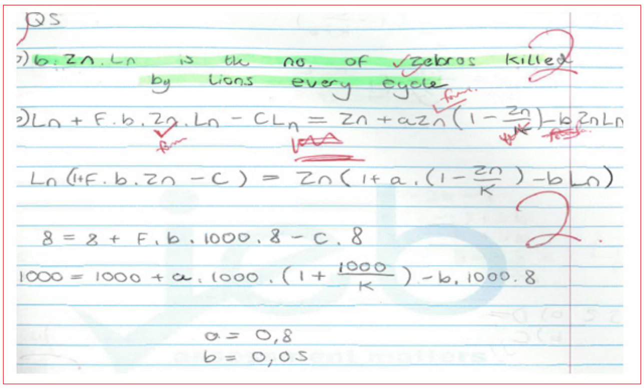


Figure 4: Learner 4 Response

Analysis of learners' responses indicated only two learners were placed in the Level 2 category. As evident in Figure 4, showed learner understood the problem by investigating the given real situation, the learner finds a real model and changes or simplifies the model, but does not know how to transfer this to a mathematical model to suit the real model. The mathematization is completely flawed. The learner proceeded by substituting values into the model that they could not structure and simplify, at that point no meaning was made in the modelling process.

Level 5 (Task 5)

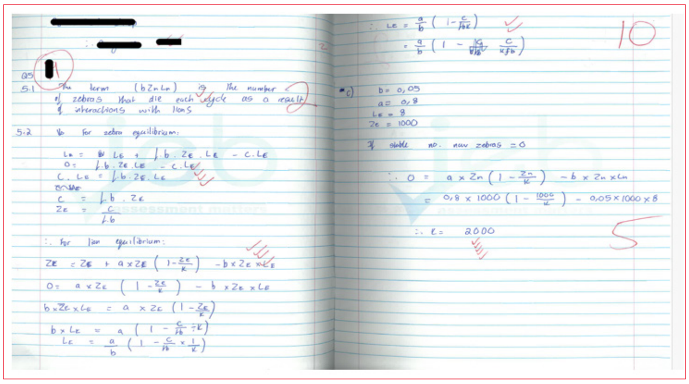


Figure 5: Learner 6 Response

The analysis showed that eight (8) learners were placed in Level 5 category. These learners demonstrated competence in executing all Levels in the modelling process relatively accurately and with precision. Figure 5 illustrates the response of one such learner. The learner was able to model the Zebra equilibrium whilst structuring and simplifying the variables correctly to produce the Zebra model and the same was done on the Lion model. The learner is able to provide a succinct explanation of mathematical concepts and models used to give a better interpretation of the mathematical results. These learners demonstrate conceptual understanding of the real model of the predator-prey Lotka-Volterra model as they build a relationship of the Zebra as $L_E = \frac{a}{b}(1 - \frac{C}{f.b.K})$

and used it to determine the carrying capacity, K = 2000. This level indicated learners succeded in translating quantities in the problem into mathematical entities and relating the mathematical entities following the information in the problem and this agrees with [2].

Level 0 (Task 4)
In this part of the discussion, we present a Task 4 phase plot depicting a predator-prey relationship between two species according to the Lotka-Volterra model. The results are indicated on the phase plot (See Task 4 on Appendix 2) with details of each sub-questions as shown in Figure 6 and others as illustrated below.

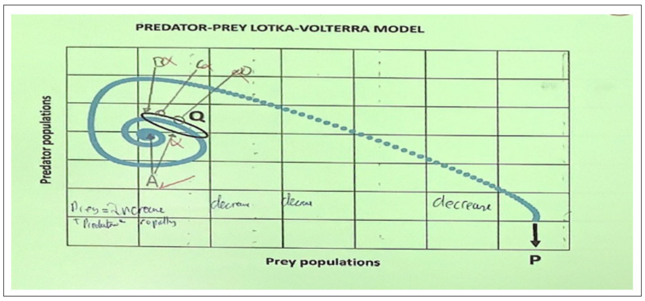


Figure 6: Response of Leaner 1

The analysis in Figure 6 was demonstrated by only one learner who could not understand the problem, the learner could not fully unpark the information and apply any meaningful ideas to the phase plot (Level 1). Only four learners could apply Level 2 in modelling the solutions on the phase plot.

Level 2 (Task 4)

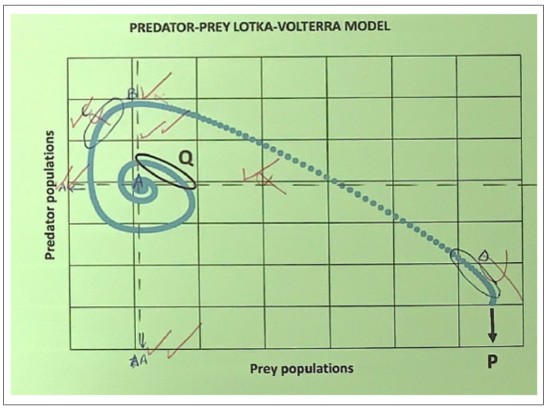


Figure 7: Response of Learner 2

The analysis in Figure 7 showed that four learners used this Level 2 category to address the problem. A real model is found by the learner, who modifies or simplifies it but cannot translate it into the exact position where it fits the real model. Partially mathematization exists in conceptualizing the relationships between the predator-prey behaviours in the phase plot.

Level 4 (Task 4)

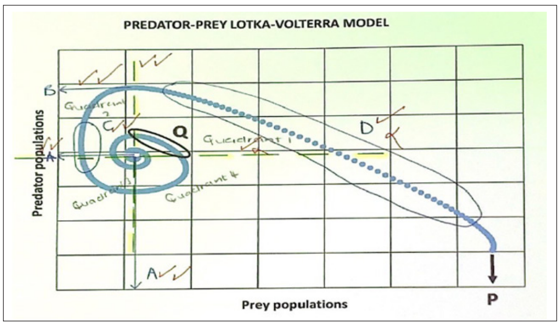


Figure 8: Response of Learner 3

Analysis in Figure 8 indicated only two learners-related results. From the provided model, the learner was able to apply and use it to formulate mathematical relationships between the two species and come up with solutions on the phase plot, the learner was also able to use dotted lines but missed given some exact locations as to whether there was a decrease or increase of any of the species relationship in respect to their population.

Level 5 (Task 4)

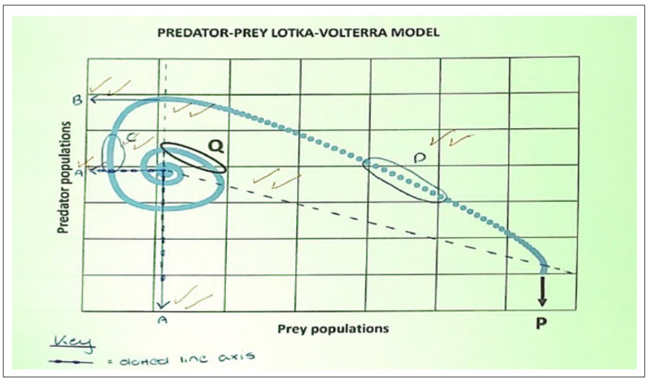


Figure 9: Response of Learner 4

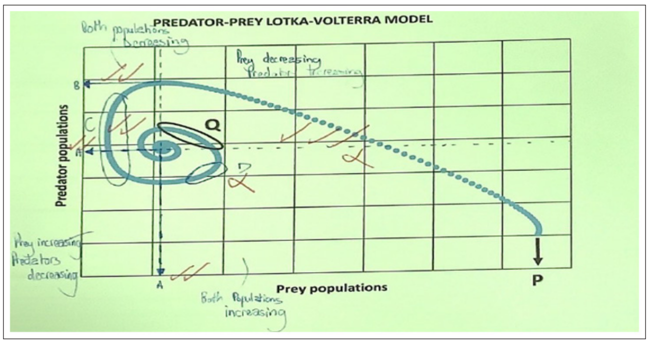


Figure 10: Response of learner 6

The analysis in Figure 9 and Figure 10 showed that eight (8) learnerss demonstrated very good competency in applying their conceptual skills in modelling the Lotka-Volterra predator-prey model. They were able to invoke their reflective and critical thinking skills and applied appropriate and relevant mathematical ideas and concepts to locate the exact points of the phase plot of the given model. In Figure 9, the learner demonstrated conceptual knowledge using the dotted lines to produce the quadrants to make their locations excellent, whilst, in Figure 10, the learner failed to locate point D correctly but rather provided a succinct explanation of mathematical concepts and models used to give a better interpretation of the model relationships results.

Conclusion and Implications
Learners could stretch their thinking and make the right decisions when given the chance to apply the modeling approach to solve a real-world problem involving Lotka-Volterra formulas or models. This allows them to produce plausible solutions that can be understood in the context of the original task. Learner progression through the various levels of the modeling framework is conceivable, as demonstrated by the responses from learners 4 and 6, who are operating at Level 5. Deconstructing the original model and adding the required parameters or suitable variables to the Zebra and Lion as Z_E and L_E, respectively, showed that these learners could read and unpark relevant information.

To obtain mathematical results, these aided in organizing and streamlining the formulae. Using the appropriate variables to create relationships and idealizing the model to enable the right mathematical representation, learner 4 was able to solve the problem successfully. Using the concept of mathematization to simplify the two relations involving the Zebra and Lion and demonstrate the Lion's carrying capacity of the variable k, as demonstrated in L_E=a/b(1-C/(f.b.K)), learner 6 was able to conceptually understand the Lotka-Volterra model. Invoking pertinent mathematical modeling expertise and strategies to solve the prey-predator problem was made possible by learner 6's ability to develop or prove the Lotka-Volterra formula. By providing distinct dotted lines on the model to the parameters of the actual problem, learner 4 was able to effectively interpret the solution. Regarding learners operating at Level 0, it is suggested that an individual's inability to comprehend and read a problem hinders them from creating a sufficient representation of the scenario, which could aid in providing structure and simplification before the mathematization process commences.

However, learners operating at Levels 1 and 2 suggest that even if a person could simplify and structure the problem, a lack of appropriate mathematical knowledge and techniques may prevent them from proceeding to the process of mathematization and mathematical working. Moreover, as demonstrated by learner 4 (see Figure 4), students may be prevented from reaching a meaningful solution even though they are aware of a strategy that could be applied to a task if the mathematical model is inadequate or if their understanding of mathematics is flawed.

In conclusion, it is our responsibility as teachers and teacher educators to make sure that reading and unparking relevant task information are regular classroom activities if we want our students to be competent and successful at solving real models involving Lotka-Volterra models using modeling strategy. Furthermore, we need to create more open-ended tasks that allow learners to choose and select appropriate mathematical models, strategies, and techniques to complete a task, as well as Lotka-Volterra formulae involving predator-prey models [24].

Appendix 1: Task 5
Problem 5
In a certain area of the African Savanna, Lions prey mainly on the Zebra population. Since a ban on poaching is strictly applied, the Lions have no predators themselves. The Lion and Zebra populations can be modelled effectively using the Lotka-Volterra recursive formulae:

$$L_{n+1} = L_n + f.\,bZ_nL_n - CL_n$$

$$Z_{n+1} = Z_n + aZ_n(1 - \frac{Z_n}{K}) - bZ_nL_n$$

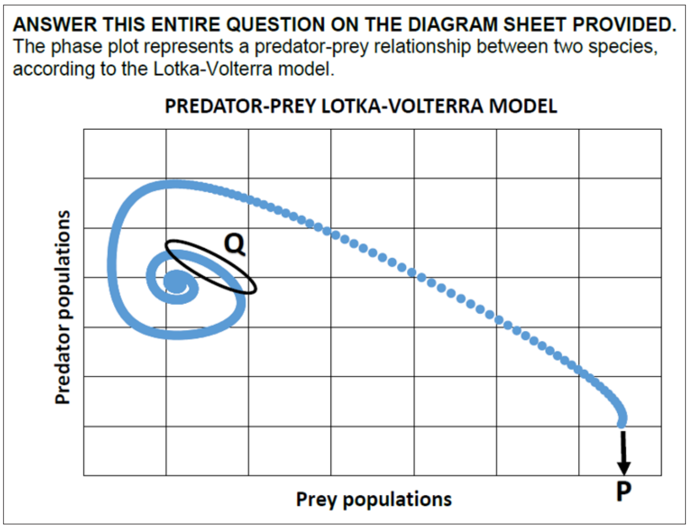
The Lions and Zebras have now reached stable populations with approximately 1000 Zebras being hunted by eight Lions. Records suggest that a Lion encountering a Zebra result in a kill occurring in 5% of the cases.

The intrinsic growth rate of the Zebra population is known to be 0,8 (80%) per annum.
a) Explain the meaning of the term $bZ_n L_n$
b) If the stable populations of Lions and Zebras are given by L_E and Z_E respectively,

Prove that: $L_E = \frac{a}{b}(1 - \frac{c}{Kfb})$

c) Hence or otherwise, determine the carrying capacity K.

Appendix 2: Task 4
Problem 4



The letter P and the arrow indicate where on the axes to read off the initial population of the prey. Do the following in a similar manner:
a) Indicate with arrows and the letter A, where on the axes to read off the equilibrium population of each species.
b) Indicate with an arrow and the letter B, where on the axis to read off the maximum population of each species.
The letter Q and the encircled region indicate where on the phase plot a decreasing prey population and an increasing predator population occurs for the second time. Do the following in a similar manner:
a) Indicate with an encircled region and the letter C, where on the phase plot the predator population is decreasing most rapidly for the first time.
b) Indicate with an encircled region and the letter D, where on the phase plot the change in the population is the greatest from one time period to the next.

Use dotted lines and draw in the axes that divide the phase plot into the four quadrants that indicate the different ways in which the population increases or decreases.

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