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Analytical Study of Random Oscillations of Vibration Protection Devices with an Elliptical Hysteresis Loop for Absorbing Harmful Energies by Non-Linear Mechanical Systems and their Elastic-Damping Elements

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ABSTRACT

The paper discusses the analysis of forced random oscillations of a non-linear mechanical system, namely a heavy drilling machine, taking into account the damping ability of the operator (human body) as a mechanical system and the elliptical hysteresis loop of the absorption of harmful energies by the elastic-damping element.

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Introduction

In modern industry (machine building, shipbuilding, aircraft building, etc.), such vibration and noise protection devices are widely spread, whose elastic-damping elements are characterized by an elliptical hysteresis loop for absorbing harmful energies. Nevertheless, the mathematical description of the hysteresis loop of the mentioned form is still not known [1]. Taking into account the damping ability of a non-linear mechanical system, namely a heavy drilling machine, the driver (human body) in it as a mechanical system, and the elliptic-shaped hysteresis loop for the absorption of harmful energies by the elastic-damping element, we proposed for the first time a complete mathematical description of such an ellipse for the analysis of forced oscillations. of the loop, which is presented in the first drawing

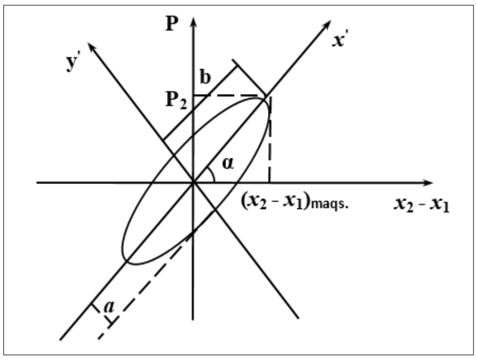

Figure 1: A mechanical model of an ellipse-shaped hysteresis loop of an elastic-damping element and its mathematical description.

$$x' = y\cos\alpha + p\sin\alpha; \;\; y' = -\sin\alpha + p\cos\alpha;$$

$$p = F_1(t); \;\; g\,\alpha = \frac{p_2}{(x_2 - x_1)_{\text{maqs.}}}; \;\; y = x_2 - x_1;$$

$$\frac{(x')^2}{a^2} + \frac{(y')^2}{b^2} = 1; \;\; y' = \pm\frac{b}{a}\sqrt{a^2 - (x')^2};$$

$$-y\sin\alpha + p\cos\alpha = \pm\frac{b}{a}\sqrt{a^2 - (y\cos\alpha + p\sin\alpha)^2};$$

$$y^2\left(a^2\sin^2 a + b^2\cos\alpha\right) + p^2\left(a^2\cos\ a + b^2\sin^2\alpha\right) +$$
$$+ 2py\sin\alpha\cdot\cos\alpha\left(b^2 - a^2\right) - a^2b^2 = 0;$$

$$P = \frac{-2y\sin\alpha\cos\alpha\left(b^2 - a^2\right) \pm \sqrt{4y^2\sin^2\alpha\cos^2\alpha \times \left(b^2 - a^2\right) - 4\left(a^2\cos^2\alpha + b^2\sin^2\alpha\right)\left[y^2\left(a^2\cos^2\alpha + b^2\cos^2\alpha\right) - a^2\cdot b^2\right]}}{2\left(a^2\cos^2\alpha + b^2\sin^2\alpha\right)};$$

$$p(y) = \frac{\beta_1 y \pm \sqrt{\beta_2 y^2 - \gamma\left(\varepsilon\, y^2 - v\right)}}{\xi},$$

where

$$\beta_1 = -2\sin\alpha\cdot\cos\alpha\left(b^2 - a^2\right);$$

$$\beta_2 = 4\sin^2\alpha\cdot\cos^2\alpha\left(b^2 - a^2\right);$$

$$\gamma = 4\left(a^2\cos^2\alpha + b^2\sin^2\alpha\right);$$

$$v = \left(a^2\cdot b^2\right);$$

$$\xi_1 = 2\left(a^2\cos^2\alpha + b^2\sin^2\alpha\right) = \frac{\gamma}{2};$$

$$\frac{dp}{dy} = \frac{\gamma}{\xi} \pm \frac{1}{\xi}\cdot\frac{\beta_1 y - \gamma\varepsilon\cdot y}{\sqrt{\beta_1 y^2 - \gamma\left(\varepsilon y^2 - v\right)}};$$

In this case, the internal resistance of the elastic-damping element is determined by the formula

$$\lambda = \lambda(\mu_2) = \frac{b^2}{a^2}\cdot g\,\alpha(\dot{x}_2 - \dot{x}_1)\mu_2^{-2};$$

It is worth noting the case when

$$tg\,\alpha = \frac{p_2}{(x_2 - x_1)_{\text{maqs.}}} < 1,\quad \lambda(\mu_2) \approx \frac{b^2}{a^2}\cdot\frac{\dot{x}_2 - \dot{x}_1}{\mu_2}\,p_2^{-1} \text{ then}$$

$$ctg\alpha = \frac{(x_2 - x_1)_{\text{maqs.}}}{F_0} < 1, \text{ from which we get that}$$

$$K_2^0 = \frac{dP}{dx} \approx \left(1 + \frac{b^2}{a^2}\right) tg\alpha\,(\dot{x}_2 - \dot{x}_1)\cdot\text{sgn}(\dot{x}_2 - \dot{x}_1).$$

of a non-linear mechanical system, namely a heavy drilling machine, a machine in it (of the human body) as a mechanical system of damping ability and taking into account the elliptic hysteresis loop of the absorption of harmful energies by the elastic-damping element, the mechanical model and the calculation scheme in our case will take the form

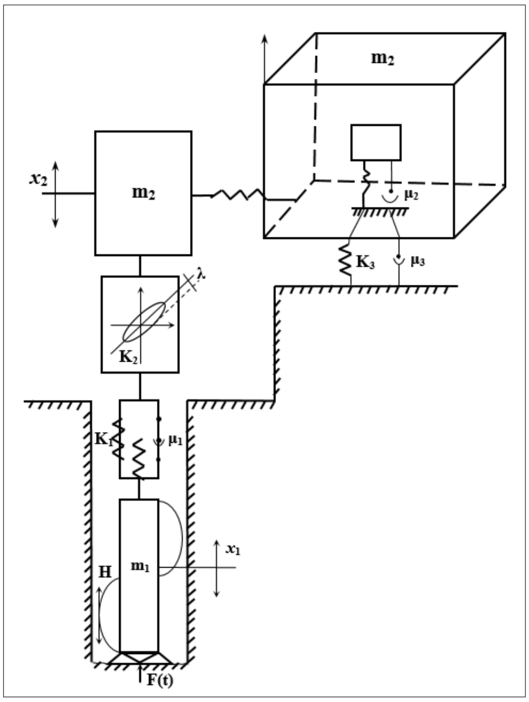

Figure 2: Considering the damping ability of the non-linear mechanical system, namely the heavy drilling machine, the driver (human body) in it as a mechanical system, and the elliptic hysteresis loop of the absorption of harmful energies by the elastic-damping element, a mechanical model and calculation scheme.

The values of the parameters given in the figure are as follows:
m_1 - the mass of the elastic-damping element of the system and the anti-vibration system
K_1 - stiffness of the elastic-damping element
μ_1 - coefficient of viscous friction of the elastic-damping element
H- force of dry friction of the vibration protection system with the environment (rocks)
K_2 – the average stiffness of the anti-vibration system
λ- coefficient of quantitative characteristic of energy absorption of forced oscillations
m_2 – the total mass of the driver and the vehicle compartment
K_3 – total stiffness of the driver and the driving unit as a mechanical model
μ_3 – the total coefficient of viscous friction of the driver and the driving unit as a mechanical model
$F(t)$ – random disturbing force
$x1$, $x2$ – coordinates of vertical movements.

Methods

In our case, the derivatives of the kinetic and potential energies of the given system will be equal

$$\Pi = 2^{-1}\left\{K_1x_1^2 + K_2(x_2 - x_1)^2\left[1 + \lambda\cdot\text{sgn}(\dot{x}_2 - \dot{x}_1)\right]\right\}$$

$$\frac{\partial T}{\partial \dot{x}_1} = m_1\dot{x}_1;\quad \frac{\partial T}{\partial \dot{x}_2} = m_1\dot{x}_2;\quad \frac{d}{dt}\left(\frac{\partial T}{\partial x_1}\right) = m_i\ddot{x}_i\ (i = 1,2);$$

$$\frac{\partial \Pi}{\partial x_1} = K_1x_1 - K_2(x_2 - x_1)\left[1 + \lambda\cdot\text{sgn}(\dot{x}_2 - \dot{x}_1)\right];$$

$$\frac{\partial \Pi}{\partial x_2} = K_2(x_2 - x_1)\left[1 + \lambda\cdot\text{sgn}(\dot{x}_2 - \dot{x}_1)\right].$$

The dissipative function of the Relay will take shape in this case

$$\Phi_1 = \frac{1}{2}\mu_1\dot{x}_1^2 + \int H\,\text{sgn}\,\dot{x}_1\,d\dot{x}_1;$$

$$\Phi_2 = -\mu_1\dot{x}_1 + H\,\text{sgn}\,\dot{x}_1,$$

Where *H* is the dry friction force [2].

Taking into account the mentioned facts, the differential equations of motion of the given system will take a general form

$$\begin{cases} m_1\ddot{x}_1 + H\,\text{sgn}\,\dot{x}_1 + \mu\dot{x}_1 + K_1x_1 - K_2^0(x_2 - x_1)\left[1 + \lambda\,\text{sgn}(\dot{x}_2 - \dot{x}_1)\right] = F(t); \\ m_2\ddot{x}_2 + K_2^0(x_2 - x_1)\left[1 + \lambda\,\text{sgn}(\dot{x}_2 - \dot{x}_1)\right] = 0. \end{cases} \quad (1)$$

Using the methods of statistical and harmonic linearization, the equations will take a general form

$$\begin{cases} m_1\ddot{x}_1 + \mu\dot{x}_1 + K_1x_1 - K_2^0(1 \pm \lambda)(x_2 - x_1) - \dfrac{4H}{\pi A_1\omega}\cdot\dot{x}_1 = F_0\cos\omega t; \quad (2) \\ m_2\ddot{x}_2 + K_2^0(1 \pm \lambda)(x_2 - x_1) = 0. \end{cases}$$

The solution of the free oscillations of the given system can be found in the following way

$$x_1 = C_1\cdot e^{st};\quad x_2 = C_2\cdot e^{st},$$

where C_1, C_2 are undefined constants; s – quality indicator. Accordingly, the characteristic equation of the system will take the form:

$$s^4 + \left(\mu_1 + \frac{4H}{\pi A_1\omega}\right)\cdot m_1^{-1}\cdot s^3 + \left[\frac{K_1 + K_2^0(1 \pm \lambda)}{m_1} + \frac{K_2^0(1 \pm \lambda)}{m_2}\right]s^2 +$$

$$+\left(\mu_1 + \frac{4H}{\pi A_1\omega}\right)K_2^0(1 \pm \lambda)\cdot m_1^{-1}m_2^{-1}s + \frac{K_1\cdot K_2^0(1 \pm \lambda)}{m_1m_2} = 0.$$

After the appropriate notation, we have:

$$a^0=1;\; a_1^0=\left(\mu_1+\frac{4H}{\pi A_1\omega}\right)\cdot m_1^{-1};\quad a_2^0=\frac{K_1+K_2^0(1\pm\lambda)}{m_1}+\frac{K_2^0(1\pm\lambda)}{m_2};$$

$$a_3^0=\left(\mu_1+\frac{4H}{\pi A_1\omega}\right)K_2^0(1\pm\lambda)\cdot m_1^{-1}m_2^{-1};\; a_4^0=\frac{K_1\cdot K_2^0(1\pm\lambda)}{m_1m_2},$$

In our case, all the coefficients of the characteristic equation are positive, and the condition for the system to operate in a stable mode will take the form

$$a_3^0\left(a_1^0a_2^0-a^0a_3^0\right)-a_4^0\cdot\left(a_1^0\right)^2=\frac{\left(\mu_1+\dfrac{4H}{\pi A_1\omega}\right)}{m_1m_2}\times\frac{(K_2^0)^2(1\pm\lambda)}{m_2^2}>0\,.$$

This means that the working mode of the research system is sustainable in all cases [3]. The general solution of the system of equations (1) will accordingly take the form:

$$x_1=x_1^{(1)}+x_1^{(2)}=e^{-p_1t}\left(A_1^{(1)}\cos q_1t+B_1^{(1)}\sin q_1t\right)+$$
$$+e^{-p_2t}\left(A_1^{(2)}\cos q_2t+B_1^{(2)}\sin q_2t\right);$$
$$x_2=x_2^{(1)}+x_1^{(2)}=e^{-p_1t}\left(A_2^{(1)}\cos q_1t+B_2^{(1)}\sin q_1t\right)+$$
$$+e^{-p_2t}\left(A_2^{(2)}\cos q_2t+B_1^{(2)}\sin q_2t\right),$$

Were

$$s_1=-p_1+iq_1;\; s_2=-p_1+iq_2;\; s_3=-p_2+iq_2;\; s_4=-p_2+iq_1;$$

$$A_2^{(1)}=\frac{1}{K_2^0(1\pm\lambda)}\left\{\left[m_1p_1^2-m_1q_1^2-\mu_1+\frac{4p_1H}{\pi A_1\omega}+K_1+K_2^0(1\pm\lambda)\right]A_1^{(1)}-\right.$$
$$\left.-\left[m_1p_1q_1+\mu_1+\frac{4p_1H}{\pi A_1\omega}\right]B_1^{(1)}\right\};$$

$$B_2^{(1)}=\frac{1}{K_2^0(1\pm\lambda)}\left\{\left[m_1p_1q_1-\left(\mu_1+\frac{4p_1H}{\pi A_1\omega}\right)+q_1\right]A_1^{(1)}+\right.$$
$$\left.+\left[m_1\left(p_1^2-q_1^2\right)-\left(\mu_1+\frac{4p_1H}{\pi A_1\omega}\right)p_1+K_1+K_2^0(1\pm\lambda)\right]B_1^{(1)}\right\};$$

$$A_2^{(2)}=\frac{1}{K_2^0(1\pm\lambda)}\left\{\left[m_2(p_1^2-q_1^2)-\left(\mu_1+\frac{4p_1H}{\pi A_1\omega}\right)p_2+K_1+K_2^0(1\pm\lambda)\right]A_1^{(2)}-\right.$$
$$\left.-\left[m_2p_2q_2+\left(\mu_1+\frac{4p_1H}{\pi A_1\omega}\right)q_2\right]B_1^{(2)}\right\};$$

$$B_2^{(2)}=\frac{1}{K_2^0(1\pm\lambda)}\left\{\left[m_2p_2q_2-\left(\mu_1+\frac{4p_1H}{\pi A_1\omega}\right)+q_2\right]A_1^{(2)}+\right.$$
$$\left.+\left[m_2\left(p_2^2-q_2^2\right)-\left(\mu_1+\frac{4p_1H}{\pi A_1\omega}\right)p_2+K_1+K_2^0(1\pm\lambda)\right]B_1^{(2)}\right\}.$$

From the analysis of the obtained solutions, it can be seen that the free oscillations due to internal friction are linear over time and can no longer affect the established motion of the entire system. Private solutions of system (1) can be found in the following way [4].

$$\begin{cases}\dot{x}_1=-B_1\omega\sin(\omega t+\varphi_2);\; \ddot{x}_1=-A_1\omega^2\sin(\omega t+\varphi_1);\\ \dot{x}_2=-B_1\omega\sin(\omega t+\varphi_2);\; \ddot{x}_1=-B_1\omega^2\sin(\omega t+\varphi_1).\end{cases}$$

After inserting these values into the basic equations, it is possible to determine the unknown coefficients A_1 and B_1:

$$A_1=\frac{F_0\cdot\sin\omega_2+\dfrac{4H}{\pi}\cos(\varphi_1-\varphi_2)}{[K_2^0(1\pm\lambda)-K_1]\sin(\varphi_1-\varphi_2)-m_1\omega^2\sin(\varphi_1+\varphi_2)-\mu_1\cos(\varphi_1-\varphi_2)};$$

$$\varphi_1=\varphi_2\pm\pi n;\; n=0,1,2,\ldots;\; \varphi_1=arctg\frac{B_1}{A_1};$$

$$B_1=\frac{F_0\cdot\sin\omega_2+\dfrac{4H}{\pi}\cos(\varphi_1-\varphi_2)}{\left[\dfrac{m_2\omega^2}{K_2^0(1\pm\lambda)}\right][K_2^0(1\pm\lambda)-K_1]\sin(\varphi_1-\varphi_2)-m_1\omega^2\sin(\varphi_1+\varphi_2)-\mu_1\cos(\varphi_1-\varphi_2)};$$

$$A_0=0;\; B_0=0.\tag{3}$$

The purpose of our research is to achieve A_1 and B_1 coefficients ie. Minimization of amplitudes taking into account the damping ability of the mechanical system and the adjustment of the hysteresis loop with the elliptical shape of the absorption of harmful energies by the elastic-damping element.

From the obtained equations, we could determine the coefficient of absorption of harmful energies and their damping

$$[K_2^0]=\frac{m_2\omega^2\left[\left(F_0-\frac{4H}{\pi}\right)\pm\sqrt{\left(F_0-\frac{4H}{\pi}\right)^2-F_0\left[\left(F_0-\frac{4H}{\pi}\right)\left(\frac{m_2\omega^2+\mu_1\omega}{K_1-m_1\omega^2}-2\right)-F_0\left(\frac{m_2\omega^2+\mu_1\omega}{K_1-m_1\omega^2}-1\right)\right]}\right]}{\left(F_0-\frac{4H}{\pi}\right)\left(\frac{m_2\omega^2+\mu_1\omega}{K_1-m_1\omega^2}-2\right)-F_0\left(\frac{m_2\omega^2+\mu_1\omega}{K_1-m_1\omega^2}-1\right)}.$$

$$[K_2^0]=[K_2^0(1\pm\lambda)]$$

Accordingly, it will become critical for the sustainability of the research system

$$[K_2^0]^k=\frac{m_2\omega^2\left[\left(F_0-\frac{4H}{\pi}\right)\pm\sqrt{\left(F_0-\frac{4H}{\pi}\right)^2-F_0\left[\left(F_0-\frac{4H}{\pi}\right)\left(\frac{m_2\omega^2+\mu_1\omega}{K_1-m_1\xi^2}-2\right)-F_0\left(\frac{m_2\omega^2+\mu_1\omega}{K_1-m_1\omega^2}-1\right)\right]}\right]}{\frac{4H}{\pi}\left(2-\frac{m_2\omega^2+\mu_1\omega}{K_1-m_1\omega^2}\right)-F_0}$$

$$[K_2^0]^{kr}=[K_2^0(1\pm\lambda)]^{kr}$$

We are interested only in the analysis of positive roots

$$\left[\frac{4HF_0}{\pi}m_2-m_1\left(2F_0^2+\frac{16H^2}{\pi^2}\right)\right]\omega^2+\frac{4HF_0}{\pi}\cdot\mu_1\omega+K_1\left(2F_0^2+\frac{16H^2}{\pi^2}\right)>0,$$

$$K_2^0(1\pm\lambda)>[K_2^0(1\pm\lambda)]_1^{kr},\quad \frac{d\varphi^0}{dK_2^0}>0;$$

$$K_2^0(1\pm\lambda)<[K_2^0(1\pm\lambda)]_1^{kr},\quad \frac{d\varphi^0}{dK_2^0}<0.$$

The analysis of the obtained inequalities shows that the stationary point for A1 will be the minimum point, i.e. In order to deviate from the resonant mode of the research system, it will be necessary to observe the following condition

$$K_2(1\pm\lambda)<\left[K_2(1\pm\lambda)\right]_2^{kr}.$$

For the B_1 coefficient, the anti-resonance mode is achieved when $K_2^0(1\pm\lambda)=0$, accordingly, the critical values of the absorption of harmful energies and their damping coefficients are determined by the formulas:

$$\mu_{\text{kr}} = \pm\sqrt{\frac{\frac{\left(K_1 - m_1\omega^2 - m_2\omega^2\right)\cdot\omega^2}{p_\Gamma^2} - 2\left[\left(K_1 - m_1\omega^2\right)^2 - \omega^2 K_2^o\, m_2\right]\omega^2}{2\left[\omega^4\left(K_1 - m_1\omega^2 - m_2\omega^2\right)^4\right]}} \times$$

$$\times\left(K_1 - m_1\omega^2 - m_2\omega^2\right)^2 \pm \sqrt{\frac{\left(K_1 - m_1\omega^2 - m_2\omega^2\right)\cdot\omega^2}{p_\Gamma^2}}$$

$$K_2^{\text{kr}} = \sqrt{\frac{-2\left[\left(K_1 - m_1\omega^2\right)^2\left(K_1 - m_1\omega^2\right)^2 - K_2^o m_2\omega^2\right]\cdot\omega^2\left(K_1 - m_1\omega^2 - m_2\omega^2\right)^2 +}{2\left[\omega^4\left(K_1 - m_1\omega^2 - m_2\omega^2\right)^4\right]}}$$

$$\overline{+4\omega^2\left(K_1 - m_1\omega^2 - m_2\omega^2\right)^4\cdot\left(K_2^o - m_2\omega^2\right)^2 \times}$$

$$\overline{\times\, p_\Gamma^2\left(K_1 - m_1\omega^2 - m_2\omega^2\right)^2}\;;$$

$$\frac{1}{\mu} = \frac{\nu_3}{\mu_3} + \frac{\nu_4}{\mu_4};\; \mu = \frac{\mu_3\cdot\mu_4}{\nu_3\cdot\mu_4 + \nu_4\cdot\mu_3};\; 0<\nu_3\le 0{,}7;\; 0<\nu_4\le 0{,}8.$$

It can be seen from the obtained equations that by selecting the optimal values of the absorption of harmful energies and their damping coefficients, i.e. taking into account the regulation of the hysteresis loop with a certain elliptical shape, it is possible to achieve effective vibration protection of the entire research system and, accordingly, of the driver and the vehicle part. Considering the damping ability of the non-linear mechanical system, namely the heavy drilling machine, the driver (human body) in it as a mechanical system, and the elliptically shaped optimal hysteresis loop for the absorption of harmful energies by the elastic-damping element, the amplitude-frequency characteristic, as a result of the analysis of the images obtained above, will take the form

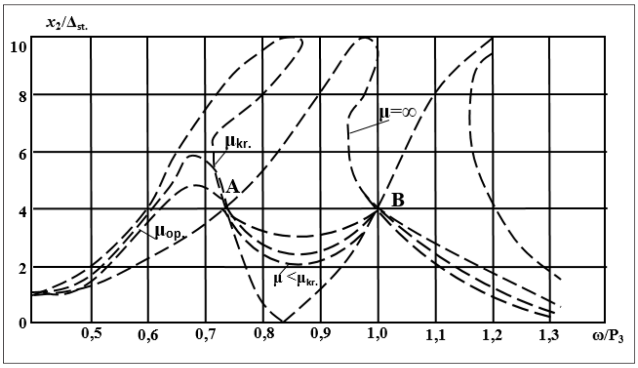

Figure 3: Amplitude-frequency characteristic of the research system and, accordingly, the driver and the driving part

Conclusion

From the analysis of the equations obtained above and Fig. 3, it can be seen that by selecting the optimal values of μ - at points A and B, which correspond to the dynamic unstable zones of the research system, the A_1 and B_1 coefficients, i.e. The values of the amplitudes are always bounded, which means that the operating mode of the research system is stable in all cases and it will be completely deviated from the resonance modes.

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