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Anisotropy of Mass and Generalization of Hamilton's Optico-Mechanical Analogy

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ABSTRACT

This article attempts to introduce the concept of mass as a tensor quantity, in contrast to the conventional view of mass as a scalar. It is shown that, in general, mass exhibits the property of anisotropy. The reduced optico-mechanical analogy of Hamilton, which can be applied to all scales of describing the motion of material bodies, is extended to the case of mass anisotropy. Examples from cosmology and condensed matter physics (quasiparticles) are considered, and a justification for such a generalization is provided.

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Subject of Study

The concept of mass is one of the most complex in modern physics. Mass acts as both a measure of inertia (a measure of resistance to external influences) and as a gravitational charge, as well as energy. Mass can rapidly change its «appearance». Attempts to conceptualize mass as something tangible, as Leibniz called "antitipy" (similar to pressure or temperature), have also been unsuccessful. The historical connection between mass and the quantity of matter (one expression of this link being the law of mass conservation formulated by Lomonosov) was also called into question after the discovery of nuclear reactions. Since, in modern field theory, matter is most often regarded as a form of the field (the mass of a nucleon consists of only a few percent of the mass of quarks, with the rest being the energy of their binding), mass becomes an extremely abstract concept [1].

Introduction

It appears that, many physical processes contribute to the formation of mass, making it impossible to point to a single cause of its origin. In the early twentieth century, multiple attempts were made to define inertia as an electromagnetic phenomenon [1]. Thomson, Heaviside, Wien, Abraham—this is by no means a complete list of scientists who explicitly discussed the concept of electromagnetic inertia. In such concepts, the charge q, is fundamental, and mass is derived, resulting from the electromagnetic interactions of the charge with its surroundings. For example, Heaviside derived the formula: $\nabla U = \frac{q^2 v^2}{3ac^2}$, by calculating the additional energy of the electromagnetic field produced by a conducting sphere of radius a, moving at velocity v. He concluded that the increase in mass due to the electromagnetic field is equal to $m = \frac{2q^2}{3ac^2}$, by comparing this result with $\frac{mv^2}{2}$. In other words, the charge acts as a center for redistributing the energy of the electromagnetic field, which we perceive as the action of inertia. The initial optimism for fully determining inertial mass in this way gradually faded, as it became clear that mass could not be entirely explained through this approach. Attempts were made to distinguish between real mass and the apparent electromagnetic addition. Abraham noted that electromagnetic mass is not a scalar but a tensor with the symmetry of a rotation ellipsoid [1].

It is also worth mentioning Föppl's attempt to develop the concept of gravitational charges with opposite signs, analogous to the positive and negative charges in Maxwell's electromagnetic field theory [1]. Föppl explained the absence of observed mass repulsion by suggesting that masses of opposite signs, due to their repulsion, separated. The masses opposite to those we observe moved to the edge of the universe and are no longer observable. However, it is evident that they influence the boundary conditions of the universe's existence.

Another significant attempt to understand the nature of mass is Mach's principle, which states that the mass of every physical object is shaped by the entire universe [2,3]. In other words, if all matter is removed from the surrounding space, a test body would either drastically reduce its mass or lose it altogether. Some uncertainty in this question arises from the internal structure of the object itself—how much the object determines its own mass. It is known that modern physics has yet to correctly define internal energy, as any attempt to determine the interaction energy of the parts of a physical body result in the divergence of the energy integral.

However, based on Mach's principle, it is evident that the anisotropy of mass distribution in the universe must result in inertial mass depending on direction. In other words, the scalar definition of mass $F = ma$ is extended to a tensor representation $F_i = m_{ij}\, a_j$ [1,3]. This tensor must be symmetric, as the angular dependence should be expressed by an even function of the angle $m(0) = m(\pi)$, otherwise, there would be abrupt changes in kinetic and potential energy. The dependence of mass on the angle can be expanded using Legendre polynomials, where the simplest anisotropic term is given by $\nabla m \sim \frac{1}{2}(3cos\theta^2 - 1)$.

Such anisotropy was investigated by Cocconi and Salpeter [1,3]. They based their research on the angular distribution of matter in the galaxy, hypothesizing that galactic anisotropy contributes significantly to the angular dependence of the mass tensor. The researchers attempted to detect this dependence using the Zeeman effect during the daily rotation of the apparatus, which would create different relative angles of the magnetic field direction toward the center of the galaxy. However, frequency splitting associated with mass differences at different times of day was not detected in their experiments, with a precision of $\frac{\nabla m}{m} < 10^{-10}$.

In the next series of experiments, the authors proposed using the Mössbauer effect, which provides good relative energy level resolution. As a result of the second series of experiments, it was shown that mass anisotropy does not exceed $\frac{\nabla m}{m} < 10^{-16}$. The negative result could either refute or support Mach's principle. As Dicke noted, all particles and fields should exhibit the same anisotropy, which could explain the negative result observed in these experiments [4]. Additionally, during Cocconi and Salpeter's time, the concept of dark matter had not yet been developed. Dark matter is now understood to play a key role in shaping inertial mass. It is clear that, considering the influence of dark matter, the anisotropy associated with the angular distribution of ordinary matter does not disappear, but its relative contribution decreases significantly.

The inability to experimentally verify these findings might be related to another factor. It is well known that there is a duality between equations and boundary conditions. It is quite possible, as Wheeler and Singh pointed out, that Mach's principle pertains more to boundary conditions and the distribution of matter at infinity rather than to the form of the equations in the general theory of relativity [3]. Based on this assumption and Cocconi and Salpeter's experiments, the distribution of matter at infinity is highly isotropic.

Ya. B. Zeldovich also indicated that the real world corresponds more closely to Lobachevskian geometry than to Euclidean geometry, a point he highlighted in a work that he did not manage to publish [5]. In this work, while studying the asymptotic behavior of the explosion process equation in cosmology (Bernoulli's equation $\frac{dx}{dt} = a(t)x^2 + b(t)x + c(t)$, where a,b,c are periodic functions), he concluded that it was necessary to redefine the phase space and replace the real axis RR with the projective line, i.e., the circle RP^1, which includes the point $x = \infty$.

A more successful attempt at defining mass as an electromagnetic substance, taking into account Mach's principle, was made by G.V. Ryazanov [6]. He applied the approach of Dirac, Feynman, and Wheeler, using advanced waves, who aimed to construct new electrodynamics. In this framework, both retarded and advanced waves equally contribute to the formation of the potential. The field created, for example, by an accelerating electron, would take the form: $E = \frac{E_r + E_a}{2}$, where E_r is the retarded field and E_a is the advanced field. As a result of his calculations, Ryazanov concluded that the electron's mass is: $m = \sqrt{N\, log\, logN}\frac{e^2}{3R}$, where N is the number of particles in the universe (according to Ryazanov's definition, the number of particles under the horizon), and R is the radius of the universe.

In modern times, the Feynman-Wheeler concept is known as the transactional theory [7,8]. Whereas standard electrodynamics only considers retarded waves coming from the past, in the transactional model, photon emission from a source is impossible without agreement from the absorber, which emits advanced waves. This can be likened to a four-dimensional resonator where time serves as the fourth dimension, leading to the emergence of the concept of nonlocal action [2].

In the context of discussing anisotropy, it is essential to mention the work of S.E. Shnoll and V.A. Panchelyuga on observing correlations between different types of histograms, separated both in space and time [9]. In Shnoll's work and that of his collaborators, solar correlations (1440 minutes) and stellar correlations (1436 minutes) were discovered. The detection of a stellar daily period indicates a dependence of the histogram shapes on the position of the observation site relative to the "sphere of fixed stars," extending the problem of study beyond the Solar System. Additionally, Shnoll's group found a sharp anisotropy in radioactive decay depending on the direction of the collimator. Since the histograms had entirely different natures and energy ranges, the only common factor, according to Shnoll, was spacetime. He believed that at every subsequent moment, its characteristics were different and changed in a wave-like manner. However, the characteristics of spacetime, in agreement with general relativity, are determined by the distribution of matter in the universe.

Since the probability density of a process (e.g., radioactive decay) is determined by the square of the alpha particle's wave function $P(\theta) = \varphi\varphi^*$, and the evolution of the wave function is described by the Schrödinger equation: $i\hbar\frac{\partial\varphi}{\partial t} = [\frac{P^2}{2m(\theta)} + U(r)]\varphi$; it becomes evident that the evolution depends on mass, suggesting that all processes, whose histograms were observed by Shnoll's group [9], are united by a functional similarity due to the common dependence of mass on the angle.

Additionally, it is necessary to highlight the works [10,11] on the consideration of anisotropy of relativistic mass, which becomes different along and perpendicular to the direction of motion at high velocities. For instance, L.B. Okun, in his fundamental article, attempted to clarify the definition of mass, pointing out that even Einstein and Feynman used different definitions of mass depending on the context. Okun argued that the only universal mass is the rest mass, which serves as a basis for further developments in the concept of mass, allowing for more complex formulations.

In this sense, a preferred reference frame emerges. Furthermore, for relativistically moving bodies, there is no single measure of inertia, as in contrast to the non-relativistic case, the acceleration is not directed along the force and depends on the relationship between velocity and force, once again illustrating anisotropy [10].

Methodology

All the concepts from the Introduction regarding mass anisotropy on different scales can be unified through the analogy applied by Hamilton when developing the mathematical framework of mechanics, as his analogy is universal. Hamilton extended Maupertuis' principle by using the model of wavefront propagation as a fundamental metaphor for describing mechanical motion. In our generalization of Hamilton's metaphor, the trajectory of a material body aligns with velocity or a light ray, along which the Umov-Poynting vector is directed, transferring energy, while momentum coincides with the gradient of the wavefront and the wave vector. In our view, Hamilton made a significant reduction in his analogy, as he disregarded anisotropy (in the general case, the wave vector and the ray vector do not coincide in direction). In optics, anisotropy is defined by the refractive index, which in turn depends on the susceptibility of the material. These properties of matter determine a certain "inertia" of the electromagnetic ray — the slowing of its phase velocity, which, when developing Hamilton's analogy, can become the anisotropy of the material "inertia" — mass. Dielectric permeability, like mass, in many cases appears to be a scalar and was long considered as such, manifesting inhomogeneity and anisotropy only at the microscopic level, which was lost in experiments measuring the average value.

The generalization of Hamilton's optico-mechanical analogy formalism, which represents mass as a tensor quantity, allows us to "stitch together" several limiting points in the theory of mass. Mass, as a function of the number of particles N, $m = f(N)$, must behave uniformly (preserving dimensionality). And since its nature (as a tensor) can be traced at the extreme points of the function $f(N)$, it cannot change as N increases. Indeed, Mach's principle shows that on galactic scales, mass should be a tensor. Solid-state theory demonstrates that the mass of an electron is also a tensor quantity and varies within a wide range, which is commonly referred to as the effective mass of the electron. The tensor nature of mass, manifesting on both macro and micro scales, should be preserved for any number of particles.

It is also necessary to highlight the foresight of Hamilton's metaphor, as it anticipated de Broglie's wave interpretation of the psi-function. Hamilton, well before the formalism of quantum mechanics, demonstrated the wave nature of the dynamics of matter, as Schrödinger pointed out [12].

Thus, one of the motivations for this article was to present a complete analogy of Hamilton and the resulting more complex relationship than p=mv between momentum space and velocity space, tangent to coordinate space. In this context, coordinate space is conjugate to momentum space, and together they form phase space.

A certain paradox of the classical approach lies in the fact that, on the one hand, there is no anisotropy, and the momenta are collinear with the velocities $p = mf(|v|)v$, which depend on coordinates,

where $f = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$. On the other hand, the momenta form a dual space to the tangent space of coordinates. The spaces of coordinates and momenta are considered independent and together form a symplectic space [13].

However, the cotangent space of momenta and the tangent space of velocities should not be identical and should differ only by a proportionality coefficient $mf(|v|)$, as might be suggested by the formula $p = mf(|v|)v$; i.e., mass should not be a scalar. In other words, mass acts as an operator that maps the tangent space to the cotangent space $p = Mv$. Thus, for kinetic energy:

$$K = \frac{vMv}{2} = \frac{vp}{2}$$

A similar construct was introduced by B. Ya. Zeldovich in [14], where the author considered a Lagrangian of general form as a bilinear function of coordinates and velocities:

$$L(x, \dot{x}, t) = -\frac{1}{2}x^T K x + \frac{1}{2}\dot{x}^T M \dot{x} + x^T \beta \dot{x}$$

It is evident that the mass matrix, which defines kinetic energy, can also describe a system of bodies in the case of their coherent motion and does not necessarily have a diagonal form. In this work, Zeldovich demonstrated the important connection between parametric resonance and impedance, which, in turn, is expressed through the mass matrix.

The introduction of the mass tensor simultaneously organizes the notation of covariant and contravariant vectors [1]. If mass is defined as a tensor m_{ij}, then the covariant momentum is $p_i = m_{ij}\frac{dx^j}{dt}$, and the fundamental law of motion can be written as $\frac{dp_i}{dt} = F_i$. The kinetic energy of a particle in these notations is an invariant $\frac{1}{2}m_{ij}\frac{dx^i}{dt}\frac{dx^j}{dt}$.

Let us consider the optical-mechanical analogy formulated by Hamilton in constructing his formalism based on the principle of least action [13]. He used Fermat's principle for the motion of rays in an optically inhomogeneous medium, where their trajectory is determined by the law of "shortest propagation time".

$$\delta\int_a^b \frac{ds}{u} = 0 \quad (1),$$

where u is the speed of light, dependent on the coordinates x,y,z [15]. Essentially, Hamilton represented the refractive index profile as a kind of force field in which light moves. In the case of light rays, we have

$$u = \frac{c}{n} \quad (2),$$

then the expression

$$\delta\int_a^b \frac{n(x,y,z)ds}{c} = \delta\int_a^b \frac{\sqrt{\varepsilon}(x,y,z)ds}{c} = 0 \quad (3),$$

defines the trajectory of motion, where n is the refractive index, and ϵϵ is the dielectric permittivity. In the case of a crystal, ε becomes a tensor, and thus, in addition to inhomogeneity, anisotropy is also present.

According to Huygens' principle, for each point q_0, where q are the generalized coordinates, it is possible to define a function $S_{q0}(q)$ as the optical path length from q_0 to q, i.e., the shortest time for light to travel from q_0 to q. The level set of the function $S(q_0)(q)$at a given t determines the wavefront, and the light rays move along the trajectory defined by the ray vector, which does not necessarily have to be perpendicular to the wavefront. On the contrary, the gradient of the function $S(q_0)(q)$ is perpendicular to the wavefront [13]:

$$\nabla S = p \quad (4)$$

At the same time, the function S represents the phase of the propagating light wave. Considering the Helmholtz equation leads to the wave propagation equation, known as the eikonal equation [16]:

$$(\nabla S)^2 = n(q)^2 \quad (5).$$

According to Huygens' principle [13], the relation
$p\acute{q}=1$ (6),
implies that the direction of p depends in a complex manner on $\acute{q}$and the refractive index.

This can be reformulated as [16]:
$sn = 1$ (7),

where s is the Umov-Poynting vector, determined from relation (7), and $n$is the vector collinear with the wave vector and equal in magnitude to the refractive index.

This expression implicitly defines a quadratic form that connects the wave and ray vectors: $(pA(n)q\breve{)} = 1$

where the matrix A depends on the refractive index. From this, one can attempt to express one vector in terms of the other and obtain the explicit relation $p = B\acute{q}$, where $B = f(A)$.

This expression corresponds to the classical formula $p = mv$.
The expression (1) in mechanics takes the form:

$$\delta \int_a^b \sqrt{2m(E - V)}\, ds \quad (8),$$

where E is the energy and V is the potential. Then, continuing Hamilton's analogy, the dielectric permittivity corresponds to the expression:

$$\frac{\varepsilon}{c^2} \rightarrow 2m(E - V) \quad (9)$$

It is easy to see that if the potential V introduces inhomogeneity, then mass may introduce anisotropy, similar to the complex structure of dielectric permittivity. One can observe that dielectric permittivity is an inert characteristic of an electromagnetic wave, slowing down its phase velocity in matter. The movement of an electromagnetic wave in matter is, in fact, the general case of such motion, since a pure vacuum is a unique situation rather than a standard one for consideration.

Let us demonstrate that such a generalization of Hamilton's optical-mechanical analogy formalism describes the concept of quasiparticles and their effective mass, which is a tensor quantity. This concept is applicable in the classical description of a conduction electron as a quasiparticle with a known dispersion relation, moving near the edge of the allowed energy band. The conditions under which the classical approach is valid and when it is permissible to not distinguish between quasi momentum and momentum are discussed in detail in [17].

The foundation of the classical approach is that the quasiparticle is treated as a particle with the Hamiltonian function:

$$H(p, r) = \varepsilon_s(p) + u(r) \quad (10)$$

Here, the kinetic energy, which typically has a quadratic dependence on momentum, is replaced by the dispersion relation $\varepsilon_s(p)$, where (s) is the number of the energy zone, and $u(r)$ is the potential. There is no external magnetic field. Near the extremum points, the function $\varepsilon_s(p)$ can be represented as:

$$\varepsilon_s(p) = \varepsilon_s(p) + \frac{1}{2}\left(\frac{\partial^2 \varepsilon_s}{\partial p_i \cdot \partial p_k}\right)_{\vec{p}=\vec{p}_0} \cdot \left(p_i - p_{i_0}\right) \cdot \left(p_k - p_{k_0}\right) \quad (11)$$

Next, we introduce a symmetric second-rank tensor:

$$\frac{1}{m_{ik}} = \left(\frac{\partial^2 \varepsilon_s}{\partial p_i \cdot \partial p_k}\right)_{p=p_0}$$

which is referred to as the effective mass tensor, and it can always be reduced to a diagonal form. Further, by defining the action function $S(\overrightarrow{r,t})$ in the standard way:

$$H = -\frac{\partial S}{\partial t}$$

$$p_i = \frac{\partial S}{\partial x_i}$$

we arrive at the Hamilton-Jacobi equation:

$$-\frac{\partial S}{\partial t} = \varepsilon_s(p_0) + \frac{1}{2m_{ik}} \cdot \left(\frac{\partial S}{\partial x_i} - p_{i_0}\right) \cdot \left(\frac{\partial S}{\partial x_k} - p_{k_0}\right) + u(r)$$

We will search for the solution of the Hamilton-Jacobi equation in the form:
$S = -Et + s_0(r)$, $s_0(r)$ is being the "reduced" action.

By bringing the effective mass tensor to a diagonal form and choosing the origin in the momentum space at the extremum point of the function $\varepsilon_s(p)$, we ultimately arrive at the equation for the "reduced action":

$$\frac{1}{m_{11}}\left(\frac{\partial s_0}{\partial x_1}\right)^2 + \frac{1}{m_{22}}\left(\frac{\partial s_0}{\partial x_2}\right)^2 + \frac{1}{m_{33}}\left(\frac{\partial s_0}{\partial x_3}\right)^2 = 2[E - u(r)]$$

(where m_{11}, m_{22}, m_{33} are the principal values of the tensor m_{ik}), which generalizes the formalism of the optical-mechanical analogy (5):

$$(\nabla S_0)^2 = 2m[E - u(r)]$$

In the presence of an external magnetic field, it is necessary to make the standard substitution of the "kinematic" momentum p to P-e/c*A, where P is the generalized momentum and A is the vector potential.

Discussion

It is significant that the body mass tensor and the definition of its principal axes are local characteristics and depend on the coordinates, as they are determined by the anisotropy of the entire universe. Moving along a geodesic, the body mass tensor will be a continuous function of position. Just as, at the micro level, the effective mass of microparticles is formed in the first approximation through interactions with a crystal. Since non-relativistic mass is an additive parameter (this does not hold in the relativistic case [10]), it is evident that as the scale of the considered object increases from the electron to the atom, the molecule, etc., a change in its principal axes will be observed, as the axes of the mass tensors of its parts may not coincide.

According to Noether's theorem, if a system (M, L), where (M) is a manifold and (L) is a Lagrangian function, admits a one-parameter group of diffeomorphisms, then the corresponding Lagrange system of equations has a first integral. In our case, three principal axes and three parameters appear along which translations are allowed [13]. Accordingly, momentum is conserved along these axes. Similarly, rotations around these axes correspond to the conservation of angular momentum. Noether's theorems themselves linearize real motion; in other words, they are valid for a closed system. In reality, all systems are open, including the entire universe, which is evident from its expansion and the "reddening" of photons, indicating a local violation of the conservation of energy law.

In the classical variant, the linearization of the law of motion is expressed in that the mass matrix is unitary, whereas in reality it is very close to being unitary. In diagonal form, the mass tensor is given by:

$$m_{ii}=m(1+\varepsilon_1(r)0001+\varepsilon_2(r)0001+\varepsilon_3(r))$$

where ε_1 ε_2 ε_3 are small corrections that define the anisotropy, and their dependence on the radius vector r demonstrates the spatial dependence of the principal axes of the mass tensor.

Similarly, the mass of the electron acquires a significant correction only in a crystal, where anisotropy is significant. Likewise, the mass of a body near a black hole will be different than that far from it.

Conclusions

By expanding Hamilton's optical analogy, where, in general, the velocity vector and the momentum vector do not coincide, it has been demonstrated that mass is a tensor. The tangent space of velocities and the cotangent space of momenta do not coincide.

The second fact that verifies this assertion is the anisotropy of mass at the limiting points, when it is either too small—such as in the case of the electron—or too large—such as in cosmological-scale processes.

In the introduction of the article, we demonstrated that the idea of mass as a tensor has historically been approached by science from two perspectives: one stemming from the cosmological principle of Mach, which postulates the formation of inertial mass by all the matter distributed throughout the universe, and the other from the idea of effective tensor mass of the electron moving in a crystal. Essentially, any particle is a quasiparticle, as empty space is an abstraction. On one hand, the tensor mass of the electron arises in the field of a periodic crystalline lattice and, consequently, is not a general case for the propagation of a material body. On the other hand, Lifshitz noted that kinetic phenomena in amorphous bodies often retain the same characteristics as those in crystalline bodies. One can attempt to represent the scattering of a proton on periodic nanostructures, whose mass significantly exceeds that of the proton, and consequently discuss the effective mass of the proton. At the next level, one can envision the motion of planets that are not bound to any star and are, therefore, situated in some modulated gravitational potential (analogous to a conduction band). In this case, it may be possible to detect anisotropy between force and acceleration in the dynamics of such planets. Remarkably, in 2023, the James Webb Space Telescope discovered that solitary planets are abundant in interstellar space, where such modulation in the calculation of their motion trajectories may already manifest at the first order.

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References

1. Jammer M (1967) The Concept of Mass in Classical and Modern Physics. Moscow: Progress. Available at: https://books.google.co.in/books/about/Concepts_of_Mass_in_Classical_and_Modern.html?id=lYvz0_8aGsMC&redir_esc=y.
2. Vladimirov YuS (2017) Metaphysics and Fundamental Physics. Book 2: Three Dualistic Paradigms of the 20th Century. Moscow: LENAND: 248.
3. Tzu H, Hoffman V (1965) Gravitation and Relativity. Moscow. Mir: 544.
4. Dicke RH (1961) Experimental Tests of Mach's Principle. Phys Rev Lett 7: 359.
5. Arnold VI (2016) Mathematical Understanding of Nature. Moscow. MCNMO: 144.
6. Ryazanov GV (1993) The Path to New Meanings. Moscow. Gnosis: 368.
7. Kastner RE (2020) The Relativistic Transactional Interpretation and the Quantum Direct-Action Theory. Foundations of Physics Group. Available at: https://arxiv.org/abs/2101.00712.
8. Schlatter A, Kastner RE (2020) Gravity from Transactions: Fulfilling the Entropic Gravity Program. Journal of Physics Communications 7: 2-16.
9. Shnol SE (2007) Cosmophysical Nature of the Idea of Histogram Shapes Constructed from Measurements of Different Processes. Metaphysics. 21st Century Almanac. Issue 2: Collection of Articles (Ed. Yu. S. Vladimirov). Moscow. BINOM. KnowledgeLaboratory: 284-320.
10. Okun LB (1989) The Concept of Mass. Physics-Uspekhi 158: 511-530.
11. Bogoslovsky GY (1983) The Relativistic Tensor of Inertial Mass. MoscowUniversityBulletin, Series 3. Physics. Astronomy 24: 70-71.
12. Schrödinger E (2013) Selected Works on Quantum Mechanics. Moscow: On-Demand Publishing: 422.
13. Arnold VI (1989) Mathematical Methods of Classical Mechanics. Moscow. Nauka. Main Editorial Office of Physical and Mathematical Literature: 472.
14. Zeldovich BYa (2008) Impedance and Parametric Excitation of Oscillators. Physics-Uspekhi 178: 489-510.
15. Schrödinger E (2018) Four Lectures on Wave Mechanics. Moscow. LENAND: 48.

16. Vinogradova MB, Rudenko OV, Sukhorukov AP (1990) Theory of Waves. Moscow: Nauka. Main Editorial Office of Physical and Mathematical Literature: 432.
17. Lifshitz IM, Azbel MYa, Kaganov MI (1971) Electron Theory of Metals. Moscow: Nauka. Main Editorial Office of Physical and Mathematical Literature: 416.