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Electron Diffraction: An Essay Solely based on Classical Electromagnetic Theory

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ABSTRACT

This paper discusses up to what point the fundamental two slits electron diffraction experiment can be modelled as a purely classical electromagnetic scattering problem [1]. The trajectories of single point charges (electrons with no spin) are computed as they pass, one by one, between two ungrounded perfectly conducting spheres, all computations being exclusively based on classical electromagnetic theory [2]. The computed particles trajectories clearly exhibit "interference like" behavior like those predicted by quantum theory, namely

- Particles trajectories deviations depending on the accelerating voltage, spheres radii, separation (gap) and initial particle coordinates
- Trajectories exhibiting velocity/energy discontinuities similar to quantum trajectories
- Depending on the overall system dimensions, the energy jumps associated to observed trajectory discontinuity have been found to bear a clear association to the energy jumps in quantum systems as expressed in terms of Pank´s constant *h*.

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Introduction

The two slits electron diffraction experiment is considered as one of the *touch stones* of quantum theory, the basic reasoning being: if electrons are assumed to pass, *one by one*, through one narrow slit at a time, how could it possibly exhibit a diffraction pattern depending on the existence or not of a second slit? [1]. The quantum theory explanation is that the probability wave function describing this set up depends on the total structure and therefore it considers the existence or not of a second slit.

However, as seen from a classical point of view, a different but equivalent interpretation is also possible: as the electron approaches the two slits on a real material screen (regardless of what kind of material), how the scattered electromagnetic fields, which clearly and always depend on the slits (one, two or more, besides their shape) and the screen material, will alter the trajectory of the incoming charge? Stated this way, it is clear the trajectory of the incoming charge, even according to classical electromagnetic theory, will always be influenced by all slits present on the screen, regardless which one it may finally go through, provided it does not crash on the material plane.

Therefore, the initial purpose of the present work, solely based on classical electromagnetic theory, was to provide a *quantitative* answer to the fundamental question: are the classical electromagnetic fields scattered by the screen with two slits or, as in the present work by two metallic spheres, capable of causing electron diffraction patterns? How these patterns compare to those experimentally obtained on the classical electron diffraction experiment? Clearly, a negative answer should reassure the quantum argument: only *quantum physics interpretation* could explain particle diffraction as in the two slits experiment.

Surprisingly though, as described in the following sections, not only the classical scattered field can consistently alter the trajectory of each approaching charge, even when launched one by one but, as discussed in section 3, it can also cause them to exhibit sudden discontinuities, both in their direction but also in their kinetic energy. Furthermore, and more important, such discontinuities were found to be closely related to the simulated setup dimensions expressed in terms of *de Broglie*'s wavelength from which estimates of Plank's constant could be derived.

Modeling Details

Two equivalent modelling schemes were considered at the onset of this research

- The original set up, consisting of a large conducting plane with two narrow slits, Figure 1, which could be simulated, for example, using Basinet's principle [2] or,
- Two metallic spheres, not grounded, with a gap in-between through which, the electrons should pass, Figure 2.

Numerical modelling based on classical electromagnetic theory to be used in both cases.

For simplicity though, the second simulation scheme, figure 2, was implemented first and is described in the present work, the configuration with two slits being postponed to a future occasion.

Figure 2 describes the overall scenario considered in the present work: an electron gun with accelerating potential V launches electrons, one at a time, with an initial velocity "*v*" perpendicularly oriented towards a distant imaginary plane containing the centers of two identical perfectly conducting spheres with radii "*R*", in free space; let "*g*" denote half the *gap* between spheres which are assumed to be ungrounded and with zero net charges each.

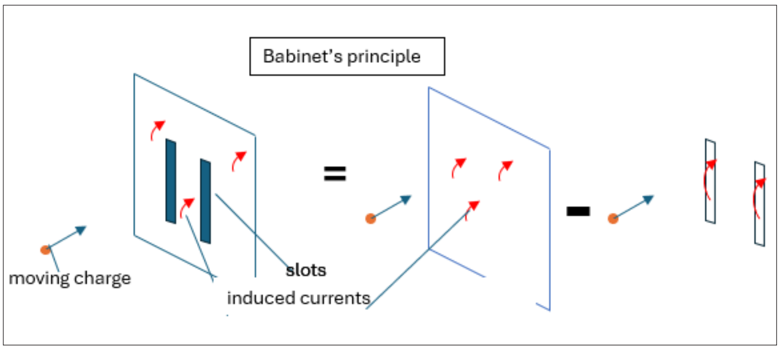


Figure 1: Proposed Configuration for Numerically Modelling the Original Diffraction Experiment.

For further reference, the resulting charge trajectory will be divided into three segments, figure 2

- **BS:** Before Spheres: starting at the electron gun till the immediate border of the spheres,
- **IS:** Inter Spheres: portion of the trajectories in between the spheres, and
- **PS:** Trajectory Past the Spheres.

Interactions between the moving charge and the spheres were accounted for, solely based on the corresponding electrostatic images, Figure 3, induced on the spheres; propagation delays were neglected since, for $V = 6$ kV results $v << c$, where c denotes the light speed in vacuum [3]. Coupling between the two spheres were accounted for solely through their interactions with the moving charge (no direct interaction between spheres considered).

Due to interaction with the induced charges on the spheres, each electron trajectory will be continuously altered, particularly while passing in between spheres (IS area, Figure 2); magnitudes and positions of the induced charge on the spheres, Figure 3, were assumed to be given by the corresponding electrostatic equations between the moving electron and the spheres being considered. No direct interaction between spheres was considered aside from that occurring through the moving charge.

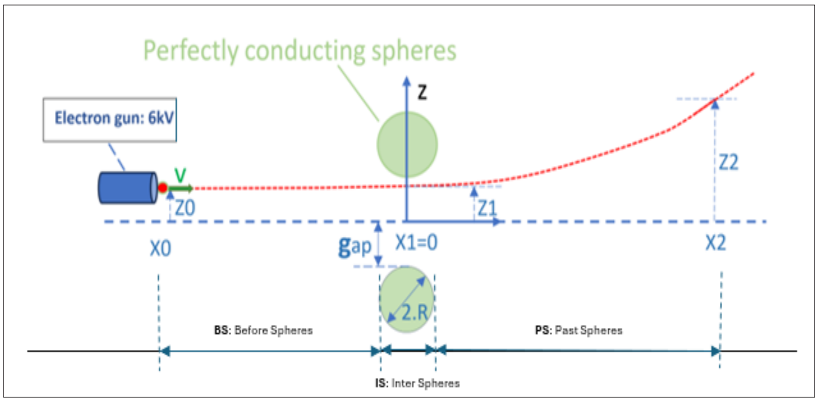


Figure 2: Electron Diffraction due to Interaction with Two Metallic Spheres.

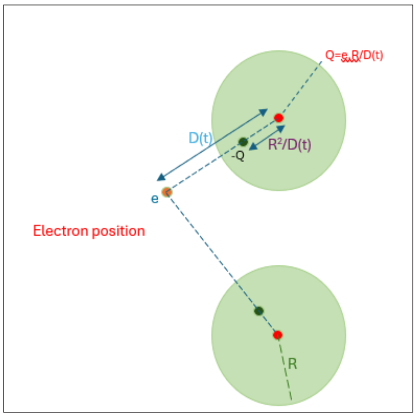


Figure 3: Moving Charge and Corresponding Induced Electrostatic Images.

2.1 Computational Modelling

For the computation modelling, the following assumptions were assumed

- Relativistic effects were neglected since only particle speeds $v << c$ was considered.
- Electrons, one at a time, are assumed to leave the electron gun, Figure 2, parallel to the "*X*" axis, with speed "*v*" in accordance with

$v=\sqrt{(2Ve/m)}$ [m/s] where

e = electron charge=1.602x10^{-31} [C]
m = electron rest mass = 9.1x10^{-31} [kg]
V = electron acceleration voltage = 6 kV

De Broglie's wavelength, here denoted as *ldb*: adopted as unit length; as described in Section 4, it turned out to be a convenient choice since many electron trajectory properties were found to be closely related to *ldb*.

$ldb=h/m\,v$ [m] where

h = Plank 's constant=6.63x10^{-34} [J. Hz^{-1}]

Forces acting on the moving electron:

F=q.E where

E = total electric field corresponding to the 4 electron images on the spheres; propagation delays were neglected ($v<<c$).

- Electron trajectory in accordance with Coulomb´s forces and Newton´s Law, magnetic forces being neglected.
- Trajectories computation: referring to Figure 2, X0 has been chosen so that distances from the electron gun and the spheres were sufficiently large according to the far-field antenna criteria, taking ldb as a reference, namely

FF= far field limit = 2.D^2/*ldb* provided D>60*ldb*.
for "*g*" and "*R*" as shown in Figure 2

- Programming: all computations and the resulting graphics outputs were performed in dedicated software based on Mathcad-14.
- Very important: all computations were conducted with 17 decimal places since most "quantum – like – effects" were "hidden" beyond the 10th decimal place.

The program used on these computations can be made available upon request to the author: please enter https://forms.gle/L9F73QKAUEeBEdgv5 and provide the requested information.

3. Numerical Results

On the elaboration of this paper, hundreds, if not more, different setup configurations, Figure 2, were programmed and evaluated

- Spheres radii and gaps between spheres ranging from *ldb/100* up to 20*ldb,*
- Several electron gun coordinates such as $0<Z0<g$, for $X0=-FF$ (Far Field limit, Figure 2).
- Field computation for 10^4 up to 10^6-time steps along trajectories, for $-FF<X<FF$.

Despite minor changes in the overall particle behavior along different trajectories, their main characteristics remained pretty much similar. This section describes some typical and more relevant results namely

- Electron trajectory deviation as it passes between the spheres
- The occurrence of "jumps" on the particle velocity, and
- The relation between the occurrence of the "jumps", electron velocity and *ldb*.

To illustrate such typical behavior the following case is detailed below in Section 3.2:

- Spheres radii and gap, Figure 2: $g=R=5ldb$
- Far field limit, Figure 2: $FF=800ldb$
- Number of points along each trajectory: 10^5 points for showing overall trajectory characteristics, Figure 4, up to 10^6 points for showing minute details (quantic like "jumps") along the trajectories as in Figures 5 onwards.
- Initial velocity: a voltage acceleration of 6 kV thus resulting in the initial electron velocity *v*=4.596 x 10^7 m/s (<< *c*) thus leading to neglecting propagation delays and relativistic effects. It´s important to note that quite similar results were also obtained for accelerating voltages V=1.5 kV and 24.0 kV with the geometrical parameters, Figure 2, properly scaled in terms of the corresponding *ldb*; this fact by itself, was considered as indicative of a deeper relation between the structure dimensions and "quantum like" particle behavior similarly to those described in Section 3.2.

3.1 Trajectories for 3 Selected Electron Gun Heights, Z0, from Central Axis: this section describes the characteristics of the electron trajectories under the conditions described above, for electrons launched from three different distances from the *Z* axis.

3.1.1 First Results Indicating a Diffraction Behavior: in this section, except stated otherwise, electron trajectories were computed for equal and consecutive 10^6 time steps although not necessarily leading to equal space steps since electron velocity changed along the path due to interaction with the metallic spheres.

Figure 4 shows trajectories for electrons launched *one-by-one* at three different heights above the X´s axis: *Z0=gap/5, gap/2* and gap/1.5, with the X and Z axis normalized to the far field distance *FF* and the spheres radii *R* respectively, Figure 2 – as shown in Figure 4, it calls attention the fact that the drawing representing the upper sphere (green line) became a thin vertical segment, since for this case *R<<FF* and *gap<<FF.*

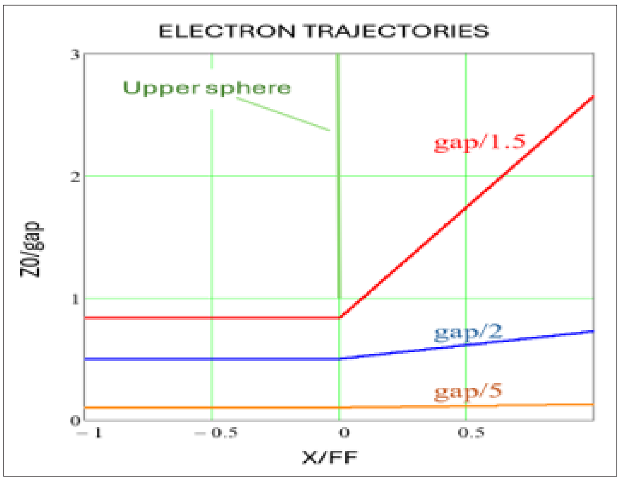


Figure 4: Electron trajectories for electrons launched at three different distances from central axis: 1/5th, 1/2nd and 1/1.5th from the central line between spheres; $g=R=5ldb$

The results described above can be considered, by itself, as a clear sign that classical electromagnetic theory can explain, at least some "***diffraction like***" phenomena, namely, fields scattered by the spheres were capable of significantly altering the trajectories of incoming charges.

As expected, Figure 4, the greater the distance the electron gun is from the X's axis, i.e., the closer the moving electron is to the upper sphere, the greater are the attraction forces from the induced image charges on the spheres consequently the greater the particle deviation while passing close by.

However a closer observation into the fine details of the charge trajectories revealed an unexpected and deeper relation with quantum phenomena: as described in sections 3.2.2 and 3.2.3, the results obtained for all simulated cases indicated the existence of a direct relation between the dimensions of the scattering structure (R, gap), the initial particle velocity "v" (and the associated ldb) and the occurrence of discontinuities, i.e. jumps, on the particle velocity (direction and modulus and therefore in the particle energy) along the trajectories.

3.2 Further investigations into the electron trajectories:

On this section only the results for *Z0=gap/2* are commented since the other two cases discussed above were found to yield very similar conclusions.

Figure 4 can be considered as a "bird´s eye" view of the three trajectories with lines appearing as perfectly smooth segments except, as expected, for sudden breaks as they pass closer to the upper sphere where more intense vertical pulls occur. However, as described below, a closer look into the trajectories data plotted in Figure 4 revealed some hidden and important details.

Figures 5a shows an initial portion of the same trajectory already shown in Figure 4 for *Z0=gap/2* but now plotted on a **very *expanded vertical scale.***

As indicated, as the electrons start drifting away from the electron gun, no noticeable changes occur up to approximately halfway to the spheres (~*FF*/1.7), where a sequence of discontinuities start showing up, both in orientation (dark point line segments added to figure 5a to help visualization), but also in velocity moduli, Figure 5b, thus raising questions as for the possible causes of such discontinuities.

The "natural suspect" for such discontinuities pointed to possible limitations - such as accuracy - on the numerical processing. To demean such hypothesis, a careful analysis was carried out on the output numerical data: an examination of the printouts of the raw data plotted in Figure 5a, b and c, indicated that the trajectory ordinates (*Z* coordinate) along each horizontal segment remained numerically constant up to the 18th decimal place whereas jumps between segments occurred between the 7th and the 11th decimal places. Significant effort to detect intermediate values between successive segment jumps has also failed so far: therefore, however intriguing it may be, the "jumpy" behavior of the graphs could not be explained with basis on "round off error" in the numerical processing. On the other hand, the coherence between results described hereby with those predicted by quantum physics strongly suggests the "hidden" causes of these discontinuities may well be due to very weak fields scattered by the spheres thus requiring quite accurate evaluation of space and time derivatives of the scattered fields which, by its turn, disturbs the original inertial particle trajectories launched by the electron gun.

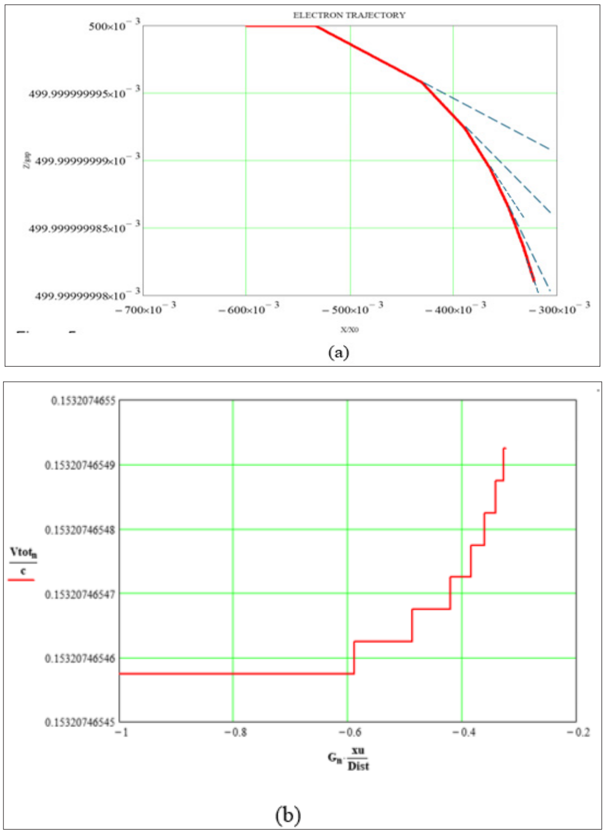


Figure 5: Electron trajectory (Z0=gap/2 in Figures 4)-attention to the expanded vertical scales: a-electron trajectory along the BS section (Figure 2), b-corresponding velocity

modulus; *g=R=5ldb*

Figures 6a and 6b correspond to the electron trajectory as it passes under the spheres, *IS* area in Figure 2: as expected, attraction forces from the image charges in the upper sphere, particularly the one with opposite charge and closer to the incoming electron, causes the charge to deviate upwards: due to proximity with the image charges on the spheres the relatively strong forces acting on the electron makes it difficult to visualize any more subtle "jumpy" behavior.

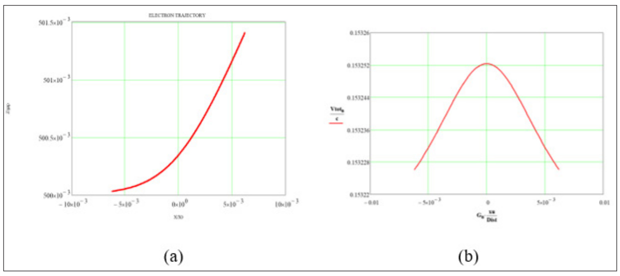


Figure 6: Electron Trajectory (Z0=gap/2 in Figures 4)-attention to the expanded vertical scales,
a-electron trajectory along the BS section (Figure 2),
b-corresponding velocity modulus; *g=R=5ldb*

Finally, as the charge moves to the *PS* region past the spheres (Figure 2) the influence of the image charges on the spheres are continuously reduced, Figures 7a and 7b, and the electron trajectory gradually becomes a straight line although some jumpy velocity behavior remains visible as shown in figure 7b.

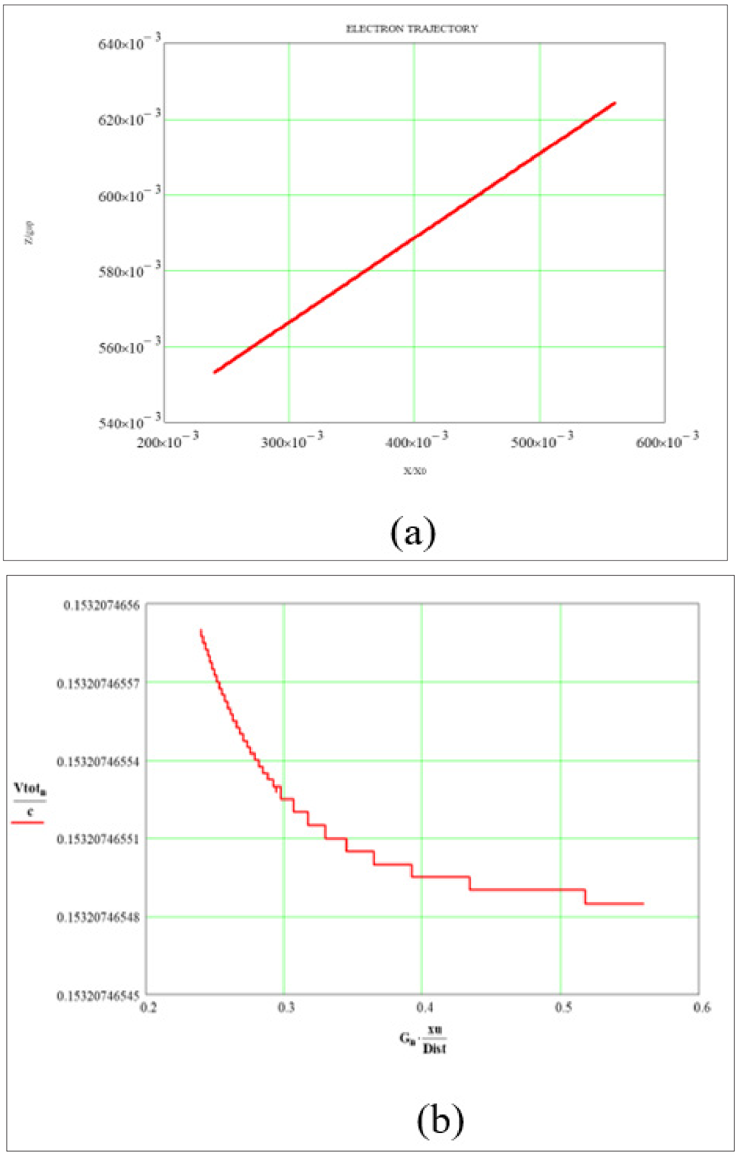


Figure 7: Electron trajectory (Z0=gap/2 in Figures 4)
a-electron trajectory along PS section (Figure 2),
b-corresponding velocity modules; *g=R=5ldb*

3.3 Electron Jumps X de Broglie's Wavelength: as shown in the previous sections, due to interaction with metallic spheres the resulting electron trajectories exhibited not only sudden changes of direction but also discontinuities/jumps on the modulus of the electron velocity, consequently on its kinetic energy.

Assuming the electron gun being sufficiently distant from the metallic spheres (Section 2.1, item *F*) the first energy jump occurs close to halfway to the spheres.

As described in the following section, the energy associated with this first jump is highly dependent on the sphere's radius and on the gap between them; in particular, as shown in the following section, for g=R it reaches a minimum value for *g=R ~* ***ldb***. Furthermore, this condition was found to hold for protons and for different accelerating voltages ranging, at least, from 1 to 24 kV.

3.3.1 Influence of (*R/ldb*) and *gap/'ldb* on the Electron Trajectories

For a given electron accelerating voltage *V* and the corresponding de Broglie's wavelength, *'ldb* let the spheres radius "*R*" and the gap "*g*" between them, Figure 2, be such that

-R=g=***a.l****db*

- 0.02<*a* <10

Let $\mathbf{h_e}$, denote an "***equivalent Plank´s constant***" defined by

$$h_e(a)=[m.e^4/dWf(a)]^{1/2}$$

where, as previously defined in section 2.1, *m* and *e* are respectively the mass and the electron charge and

dWf(a)= ***the first kinetic energy jump***

which occurs along the electron trajectory between the electron gun and the spheres.

Figures 8 shows an example of how the "***equivalent Plank´s constant***", normalized with respect to Plank´s constant ***h***

$$h(a)= h_e(a)\ /h$$

varies as a function a i.e., as a function of the sphere's radius (*R/λdb*) and the gap (*R/λdb*), for the case R=g.

As indicated, **h(a)** achieves a minimum value for

R=g~λdb

i.e. a=1.

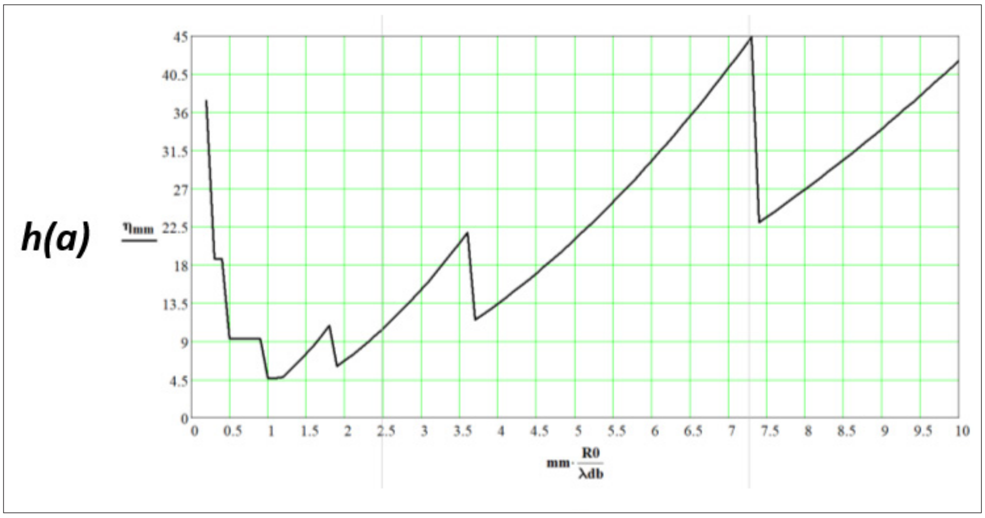
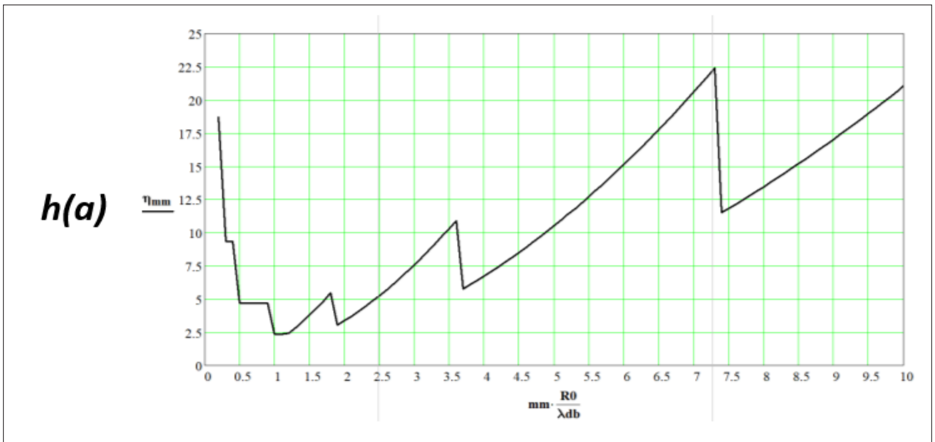


Figure 8: Variation of the Normalized Plank constant ***h(a)*** versus scaling factor ***a***= R0/λdb for accelerating voltage V=1.5kV

Quite similar results, Figures9a and 9b, were obtained for different accelerating voltages: V=6.0kV and 24 kV, as well as for protons thus corresponding to quite different λdb 's.



(a)

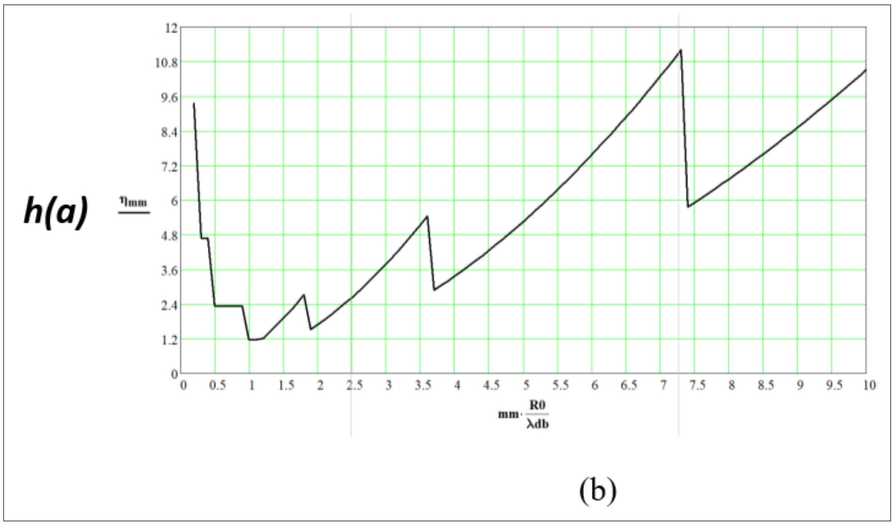


Figure 9: Variation of the Normalized Plank Constant ***h(a)*** versus Scaling Factor ***a*** for various Accelerating Voltages: a-6.0kV; b-V=24kV.

The results described above indicate that, not only can classical electromagnetic theory predict diffraction effects along particle trajectories but also and most relevant, theses occurrences are clearly dependent on the structure dimensions expressed in terms of the correspondent de Broglie's wavelength *l*db and, consequently, in terms of Plank´s constant *h*.

Conclusions

This paper describes some investigations, solely based on numerical modelling, on how the trajectory of electrical particles can be affected by the electromagnetic fields scattered from nearby objects such as metallic spheres. However, and most importantly, all calculations were exclusively based on classical electromagnetic theory without invoking nor resourcing any principle, equations or results derived from quantum mechanics. Nonetheless, as described above in section 3, deeply "hidden" inside the results based exclusively on classical, linear Maxwell's equations and Newton's law, a typical quantum behavior emerged: electron trajectories exhibiting "quantum like" jumps whenever interacting with two metallic spheres! Not only that but perhaps even most surprisingly: the occurrence of these jumps being related to the overall dimensions of the scattering structure expressed in terms of de Broglie's wavelength.

Initially, resonance effects excited by the passing charge were thought to explain the odd and discontinuous trajectories, but they could not explain the relation to de Broglie's wavelength. The same correlation to de Broglie's wavelength has been verified in very similar results when applied to heavier particles like protons as well as different accelerating voltages.

Direct relation between quantum physics and classical electromagnetic fields have already been considered by other authors [4]: based on purely analytical considerations applied to very large (universal) scenarios, the authors were able to derive the value of *h*.

In conclusion, the findings described in this paper suggest that quantum physics, or at least some of its fundaments, are deeply rooted inside classical Maxwell's equations, this being one possible reason why it could only be reached whenever calculations were performed with a great degree of accuracy, i.e., in the present case with large number of decimal places.

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