ISSN: 2754-4753

Journal of Physics & Optics Sciences

Review Article

General Resolution of the Fundamental Equation of the Global Nuclear Collision Model

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ABSTRACT

The resolution of the fundamental mechanism's equation E-F of the global nuclear collisions model is performed. It is divided into two stages. The first one, concerns the research of the one parameter minimal group which is given by applying successively conservation laws of charge and mass numbers and total energy. The second one is the determination of the possible mechanisms and their corresponding colliding system final states.

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Received: November 06, 2024; **Accepted:** November 18, 2024; **Published:** December 09, 2024

Keywords: Fundamental Equation, Mechanism-Function, Conservation-Laws, Mechanisms, Outcomes

Introduction

In the paper [2], it is demonstrated that the solutions of the F-E, that is really a system of four equations, forms an abelian group and the application of a constraint (for example a conservation law) to this equation reduces the initial group to one of its subgroups [1]. The repeated application of other constraints leads to the one parameter minimal subgroup of the initial group. Further, the application of the problem data to the equation permits to have the interval of the group parameter and consequently to have the mechanisms wanted.

The resolution of the equation of the global collision model is made in two steps, the first one consists in the application of conservation laws that have consequently the reduction of the group of solutions. The second step consists in finding the interval of the subgroup parameter and consequently the solutions (mechanisms) then the determination of the possible outcomes of the collision.

In the paper [2], the first steep of the resolution was done. That is, for a given system in collision in the initial state E and final state S and a colliding energy E, as shown in the paper the possible mechanisms are the solutions of the F-E of the model. In the first paragraph of this work, the first steep of the resolution was reconsidered because the constraint $n_1 - z_2 = n_{01} - z_{02}$ used to find the relationship between gamma and beta (parameters of the mechanism) is not a conservation law, although it has given a solution that is only an element of a very large set of solutions given by the general resolution thanks the application of total energy conservation law [2].

Fundamental Equation F-E

Effectively, the F-E is $\theta E = S$ where $\theta = G_0^{\alpha} B_0^{\beta} D_0^{\gamma} H_0^{\mu}$ and and $E = \begin{pmatrix} n_{01} & n_{02} \\ z_{01} & z_{02} \end{pmatrix}$ and $S = \begin{pmatrix} n_1 & n_2 \\ z_1 & z_2 \end{pmatrix}$. Solving it, means finding the quantities $(\alpha,\beta,\gamma,\mu)$. This is reached by applying successively neutrons, protons and total energy conservation laws to the equation. The reason of this feedback is the using in the work [2] to find a relation between gamma and beta (parameters of the subgroup obtained after the application of c_1 and c_2 conservation laws) a constraint that is not a conservation law.

The two conservation laws (proton and neutron numbers) give that $\alpha = -\beta$ and $\mu = -\gamma$.

The third constraint called c_3: $n_1 - z_2 = n_{01} - z_{02}$ in [2] will be replaced by the total energy conservation law (1), written under its relativistic form

$$M_{01}c^2 + M_{02}c^2 = M_1c^2 + M_2c^2 \quad (1)$$

Where

$$M_{0i} = m_{0i}\gamma_{0i} \quad (2)$$

$$M_i = m_i\gamma_i \quad (3)$$

m_{0i} and m_i in (2) and (3) are the rest mass of colliding and product nuclei. γ_{0i} and γ_i are the usual quantities in special relativity

$$\gamma_{0i} = \sqrt{\frac{1}{1-\frac{v_{0i}^2}{c^2}}} \quad (4)$$

$$\gamma_i = \sqrt{\frac{1}{1-\frac{v_i^2}{c^2}}} \quad (5)$$

Where, v_{0i} and v_i are successively the velocities of colliding and product nuclei in laboratory reference, $i = 1,2$ and c the speed of light in vacuum.

After replacement of (2) and (3) in (1) it comes (6)

$$m_{01}\gamma_{01} + m_{02}\gamma_{02} = m_1\gamma_1 + m_2\gamma_2 \quad (6)$$

Approximation 1: In following we consider that the nuclei linking energises are neglected, so

$$m_{0i} = n_{0i}M_n + z_{0i}M_p \quad (7)$$

$$m_i = n_iM_n + z_iM_p \quad (8)$$

Mn and Mp are successively neutron and proton mass.

The F-E (on what were applied the conservation laws c_1 $:n_{01}+n_{02}=n_1+n_2$ and c_2 $:z_{01}+z_{02}=z_1+z_2$) written as an equation system is (9)

$$\begin{cases} z_1 = z_{01} - \beta \\ z_2 = z_{02} + \beta \\ n_2 = n_{02} + \gamma \\ n_1 = n_{01} - \gamma \end{cases} \quad (9)$$

So (6) becomes

$$m_{01}\gamma_{01} + m_{02}\gamma_{02} = \left[(n_{01}-\gamma)M_n + (z_{01}-\beta)M_p\right]\gamma_1 + \left[(n_{02}-\gamma)M_n + (z_{02}-\beta)M_p\right]\gamma_2 \quad (10)$$

After a rearrangement, (10) gives gamma as a function of beta.

$$\gamma = -\beta\left(\frac{M_p}{M_n}\right) + \frac{m_{01}}{M_n}\left(\frac{\gamma_{01}+\gamma_1}{\gamma_2-\gamma_1}\right) + \frac{m_{02}}{M_n}\left(\frac{\gamma_{02}+\gamma_2}{\gamma_2-\gamma_1}\right) \quad (11)$$

Approximation 2: The formula above can be simplified further without minimizing the generality of the results by supposing $M_n = M_p$. Thus, we have

$$\gamma = -\beta + a_{01}\left(\frac{\gamma_{01}-\gamma_1}{\gamma_2-\gamma_1}\right) + a_{02}\left(\frac{\gamma_{02}-\gamma_2}{\gamma_2-\gamma_1}\right) = -\beta + f(\gamma_1,\gamma_2) \quad (12)$$

Where $a_{0i} = n_{0i} + z_{0i}$ $\quad i = 1,2$

And $f(\gamma_1,\gamma_2) = a_{01}\left(\frac{\gamma_{01}-\gamma_1}{\gamma_2-\gamma_1}\right) + a_{02}\left(\frac{\gamma_{02}-\gamma_2}{\gamma_2-\gamma_1}\right)$ (13)

As seen through the expression of f, the equality between γ_1 and γ_2 is excluded.

The last step of the resolution of the F-E is to replace Gamma by its value as a function of beta and f. It comes the following equation system

$$\begin{cases} z_1 = z_{01} - \beta \\ z_2 = z_{02} + \beta \\ n_2 = n_{02} - \beta + f \\ n_1 = n_{01} + \beta - f \end{cases} \quad (14)$$

Then, to precise the values of beta, we use the problem data ($n_i \geq 0$ and $z_i \geq 0$ with $i = 1,2$). From the four constraints obtained, we can deduce only tree inequalities, namely

$$(f - n_{01} \leq \beta \leq z_{01},\ f - n_{01} \leq \beta \leq f + n_{02},\ -z_{02} \leq \beta \leq f + n_{02}) \quad (15)$$

So, the interval wanted will be the intersection of these tree intervals.

But it lasts a problem that must be solved, it concerns how choosing γ_1 and γ_2 ?

Study of the Mechanism Function *f*

To response this question a study of the function f becomes pertinent. In fact, as seen on (13), on one hand this function depends on two variables γ_1 and γ_2 , on other hands, these variables are dependant each other once f fixed. Moreover, each of these variables is, by definition, a function of the velocity of the corresponding reaction product (5). And, because of the finite values of the velocities in the entrance channel and consequently in the outcome's channel, the γ_i are also framed, $1 \leq \gamma_1 \leq \gamma_1'$ and $1 \leq \gamma_2 \leq \gamma'_2$.

Effectively, if k represents a level curve (or the value of f) it comes equivalently the following equation

$$\gamma_2 = \left(\frac{k - a_{01}}{k + a_{02}}\right)\gamma_1 + \left(\frac{a_{01} + a_{02}\gamma_{02}}{k + a_{02}}\right) \quad (16)$$

Analytically, the formula (16) is a straight line in the plane (o,γ_1,γ_2). Thus, the study in question will focus on the sign of the factor

$\left(\frac{k-a_{01}}{k+a_{02}}\right)$ that can be negative, positive or null.

Because a graphical representation facilitates the study, (16) one's drawn, only intervals giving sense to the variables γ_i are considered. This strategy is followed along the study of the function f.

Case 1 $\left(\frac{k-a_{01}}{k+a_{02}}\right) < 0$

Hence the two solutions $\begin{cases} k - a_{01} > 0 \text{ and } k + a_{02} < 0 \text{ so, no solution for } k \\ or \\ k - a_{01} < 0 \text{ and } k + a_{02} > 0 \text{ so, } -a_{02} < k < a_{01} \end{cases}$

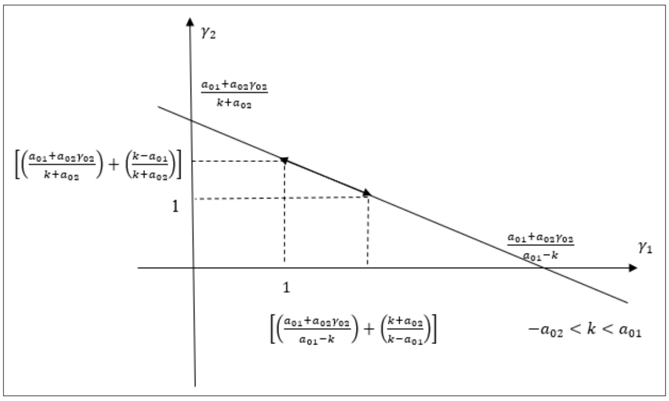

Figure 1: For determining the physical mechanisms, Because of the framing $1 \le \gamma_1 \le \gamma_1'$ of γ_1 and $1 \le \gamma_2 \le \gamma_2'$ thanks the interdependency of the variables only the part of the curve, marked by a bi rows segment, must be considered.

Analyse

As seen on the graphic (1), when γ_1 belongs to the interval $[1, \left(\frac{a_{01}+a_{02}\gamma_{02}}{a_{01}-k}\right)+\left(\frac{k+a_{02}}{k-a_{01}}\right)]$ γ_2 belongs $\left(\frac{a_{01}+a_{02}\gamma_{02}}{k+a_{02}}\right)+\left(\frac{k-a_{01}}{k+a_{02}}\right)]$ because they are interdependent. Out from these intervals, either of the variables has unpermitted values; $\gamma_i<1$. Consequently, only the part of the line retinue is that showed by the bi rows segment abovementioned. Reminding $\gamma_1 \neq \gamma_2$, we must exclude the common value

$[1+\frac{a_{02}(\gamma_{02}-1)}{a_{01}+a_{02}}]$ of the variables from the two intervals above.

$$\textbf{Case 2} \left(\frac{k-a_{01}}{k+a_{02}}\right)=0$$

$$\text{so, } k=a_{01}$$

Whatever the value of γ_1 in this interval $1 < \gamma_1 < \gamma'_1$, $\gamma_2 = \left(\frac{a_{01}+a_{02}\gamma_{02}}{a_{01}+a_{02}}\right)$

We see that γ_2 is greater than one because $\gamma_{02}>1$

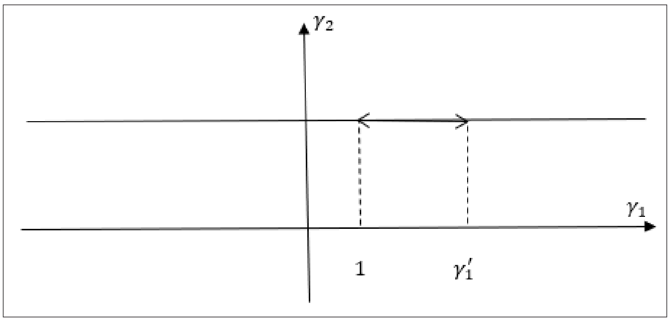

Figure 2: Because of the Framing of γ_1 and γ_2 only the Part of the Curve, marked by bi rows segment, must be considered for determining the physical mechanisms.

Note: Thanks to the relation (16), $\gamma_1 = \frac{a_{01}+a_{02}\gamma_{02}}{a_{01}+a_{02}}$ and consequently, this couple of solutions is not permissible and the value $k=a_{01}$ nor did.

$$\textbf{Case 3} \left(\frac{k-a_{01}}{k+a_{02}}\right)>0$$

We have two solutions $\begin{cases} k-a_{01}>0 \text{ and } k+a_{02}>0 \text{ so, } k>a_{01} \\ \quad or \\ k-a_{01}<0 \text{ and } k+a_{02}<0 \text{ so, } k<-a_{02} \end{cases}$

Hence the graphical representation below.

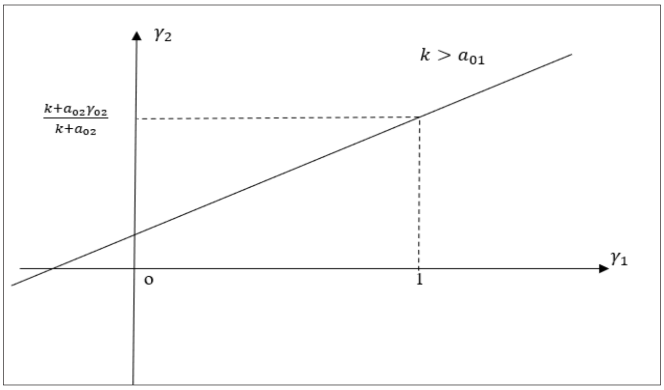

Figure 3: For Determining the Physical Mechanisms in this Case, only the Values of γ_1 and γ_2 Indicated on the Graphic are Valid. Any other Value of γ_2, gives a Value of $\gamma_1 < 1$

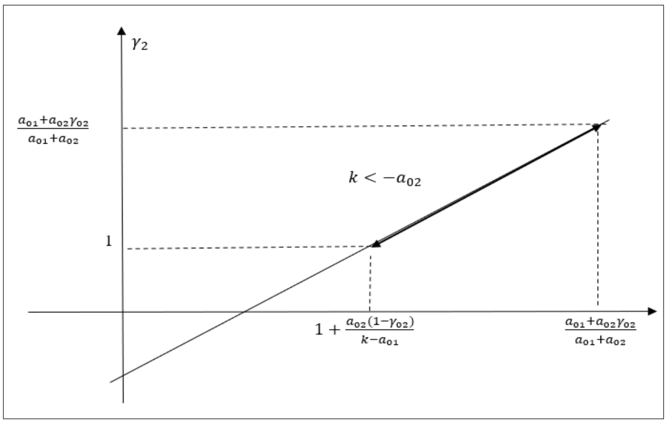

Figure 4 : For determining the physical mechanisms, because of the framing of γ_1 and γ_2 only parts of the curves, marked by bi rows segments, must be considered. The common value $\frac{a_{01}+a_{02}\gamma_{02}}{a_{01}+a_{02}}$ of the two variables must be excluded.

Second Step of the Resolution of the F – E

After this discussion, let's pursue the resolution of the F-E.

In the first paragraph, before studying the function f, we have stopped the resolution at the tree inequalities (15) defining the intervals of the parameter beta deduced by the application of the problem data ($n_i \ge 0$ and $z_i \ge 0$ with i=1,2) to the F – E.

Remark

The choice of γ_1 and γ_2 (gives according to (13) the value of k) shows that the solutions i.e. the mechanisms depend not only on nucleonic composition of the colliding nuclei and the colliding energy trough γ_{02} , but also, on the products trough they γ_i as function of their velocities. In other words, we gain in depth of the description for a given outcome, because the study above offers the possibility of calculating the relative mechanism to each couple of kinetic energies of the same product (outcome).

Mechanisms

At this stage of the resolution before any action, as in mechanics, we must give a reference because each choice done will affect the solutions.

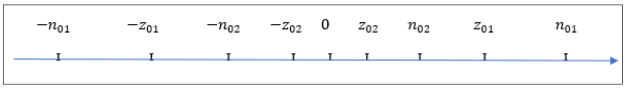

Figure 5: The choice made generally in collision experiment where $(a_{02}=n_{02}+z_{02}\leq a_{01}=n_{01}+z_{01})$ and $(z_{02}\leq n_{02}\ ,\ z_{01}\leq n_{01})$

to find the solutions, we give a value for f (for a chosen channel, the value of f is a direct consequence of a choice of a pair of product velocities we are interested in. See the paragraph II), then we find the intersection of the tree intervals (15), and finally, we write the possible mechanisms for each value of the interval retained.

Illustration, let take certain remarkable values of f, as f=0, f=a_{01} , f = -a_{02}. The last value of f is not valid because it gives an indeterminacy of γ_2 or we know that physically it has a determined value.

The value $f = a_{01}$ is not valid (see the note of the figure 2), consequently, there is no corresponding mechanism.

For the first value of f, the interval wanted is: ($-z_{02}\leq\beta\leq n_{02}$, so we have ($z_{02}+n_{02}+1=a_{02}+1$) mechanisms.

To finalize the resolution lets giving the outcomes of any mechanism we have chosen.
Thus, if we choose the mechanism $G_0^{z_{02}-1}B_0^{-z_{02}+1}D_0^{z_{02}-1}H_0^{-z_{02}+1}$ the outcome is

$$\begin{pmatrix} n_{01}-z_{02}+1 & n_{02}+z_{02}-1 \\ z_{01}+z_{02}-1 & z_{02}-z_{02}+1 \end{pmatrix} = \begin{pmatrix} n_{01}-z_{02}+1 & n_{02}+z_{02}-1 \\ z_0-1 & 1 \end{pmatrix}$$

Conclusion

The method exposed above can be summarized in seven points

- Precising the colliding nuclei i.e. giving $\begin{pmatrix} n_{01} & n_{02} \\ z_{01} & z_{02} \end{pmatrix}$.
- Writing the F-E as a system of four equations.
- Applying the tree conservation laws: c_1, c_2, c_3
- Choosing an outcome, then a couple of velocities in one of the four graphics (o,γ_1,γ_2). Thus, it is possible to calculate the value of the function f.
- choosing a reference for n_{0i} and z_{0i} , then drawing the intervals for the parameter beta, then defining their intersection (it's useful to use graphical representation).
- The retained interval gives the possible mechanisms.
- Applying each mechanism of the retained interval to the entrance channel to have the corresponding outcome.

Note: [i]From the equation linking up γ_1 and γ_2 we deduce the common value of the variables, then by a proof absurd, we prove that this value is in the intervals of the two variables.

References

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