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Method by Fundaments in Research: Review from the Commutative Algebra

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ABSTRACT

The design and creation of a model of research method revisited from the classical method, has been necessary due to arising of research on aspects of field theory and aspects that cannot be explored nor observed directly, and that requires an extension and induction of the classical method to the deep discernment and enriching analysis that comes from a true theory built through fundaments. In this paper is demonstrated the commutatively of certain diagrams and schemes of categorical objects in research whose elements are true propositions (from axioms and postulates until theorems) and whose applications between these categorical objects in research are research functores and corresponding derived morphisms.

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Introduction

The method by fundaments, is created and developed from the mathematical theory of research realized and obtained in the identification of realization invariants of a scientific research on applied sciences [1, 2, 3, 4] where the observation directly is impossible and requires an extension of the observation, the creation of a valid hypothesis to a field deduction and analysis on true propositions (already demonstrated and evident facts), the creation of a theory $Th\Sigma$, as entity fundamental and generator of true propositions and new true propositions further knowledge [1, 4, 5]. Also the experimentation, will come enforced with the true propositions as results of research operator $\mathfrak{T}$, on the set of true propositions which we denote as Φ_α, in the class α, or knowledge type (for example a knowledge class can be the belonging to a sub-branch of a study branch). The proof, will be on the theorems and experiments supported by these theorems (almost always are the true propositions obtained by $\mathfrak{T}$). Then the final law obtained is a theorem, which has more weight than a law. The theorems are eternal. A of the most important results obtained under this research theory is the flow diagram of the research methods by fundaments.

We consider the categories [6, 7] (whose points are sets of propositions) $\Phi_1, \Phi_2, \ldots, \Phi_n, \ldots$, of an Abelian category[1] $\mathcal{R}$. These sets can be set of propositions of certain class α, then these can to define the following category $\mathfrak{T}\Phi_\alpha$, whose objects are:

$$\Phi_0 \xrightarrow{d_0} \Phi_1 \xrightarrow{d_1} \Phi_2 \xrightarrow{d_2} \Phi_3 \rightarrow \ldots$$

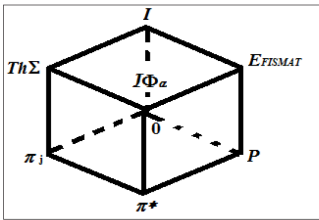
which defines the research process starting of the category of true propositions belonging to a class of the knowledge in mathematics and physics E_{FISMAT}. Also here each Φ_i, is an object of $\mathscr{R}$, and each compositions $d^i \circ d^{i+1}$, is zero. The ith −cohomology group of the complex is $H^i(\Phi_i) = \ker d^i / \operatorname{im} d^{i-1}$.

If in particular we have the categories of objects $I, E_{FISMAT}, Th\Sigma, \mathfrak{T}\Phi_\alpha, \{\pi_j\}, \pi*$, P, and 0, defined in [4, 6, 7] we have:

$$\begin{array}{ccccccccccccccc} & & & & & & & & & & \mathcal{P} & & & & \\ & & & & & & & & & & \downarrow & & & & \\ I & \rightarrow & E_{FISMAT} & \rightarrow & \Phi_\alpha & \rightarrow & \mathfrak{T}\Phi_\alpha & \rightarrow & Th\Sigma & \rightarrow & \{\pi_j\} & \rightarrow & \pi * \rightarrow & P \rightarrow 0, & (1) \\ & & & & & & & & \downarrow & & & & \downarrow & & \\ & & & & & & & & subTh\Sigma & & & & Pat & & \end{array}$$

which is totally reversible generating isomorphism between categories. For example, the theory $Th\Sigma$, can implies $\mathfrak{T}\Phi_\alpha$, in the fact that all theory considering the corresponding projections and homotopies of the parts of the flow diagram (1), these are the set of true propositions deduced to apply the research or inquiry operator $\mathfrak{T}$. The diagram (1) is an exact sequence in cohomology [8, 9].

We consider the cubic arrangement of (1), whose weight vertices are $I, E_{FISMAT}, Th\Sigma, \mathfrak{T}\Phi_\alpha, \{\pi_j\}, \pi*, P$, and 0, only. Then we have:



(2)

[1]This can be a modules category on a ring, or category of sheaves of Abelan groups on a topological space, for example topological groups.

which is commutative [10, 11, 12].

Theorem (F. Bulnes) 1. 1. The flow diagram of the research method by fundaments is isomorphic to the commutative scheme (2).

Proof. Due that the flow diagram (1) is completely reversible then conforms commutative squares. Even if we consider the additional morphisms given in the triangles (see the figure 1) $\mathfrak{T}\Phi\alpha \rightarrow Th\Sigma \longrightarrow \mathrm{sub}Th\Sigma$, $\mathcal{P} \longrightarrow \pi * \longrightarrow$ P, and $\pi * \longrightarrow P \rightarrow Pat$, these are commutatives. Then all mapping between the cube (2) and flow diagram (1) are bijective. Then the cube (2) is isomorphic to (1).

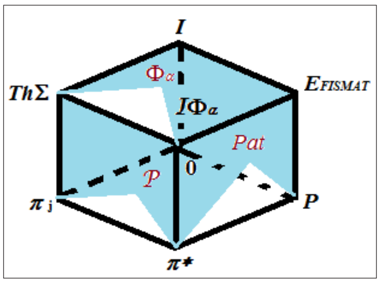


Figure1: Cubic Scheme with Commutative Triangles [11, 12].

Lemma 1. 1. $\mathfrak{T}\Phi\alpha$,is a derived category.

Proof. The morphisms $\Phi^{\bullet} \rightarrow \Psi^{\bullet} \in \mathfrak{T}\Phi_{\alpha}$, $\forall\, \Phi, \Psi \in \mathcal{R}$, are morphisms defined until the homotopy (is the category of homotopy of $K(R)$[2] with respect weak equivalences[3]), for example the application $\mathfrak{T}\Phi_{\alpha} \rightarrow Th\Sigma$, has homotopy $\{\pi_j\} \rightarrow \pi *$. Its inverse

mappings of these mappings are quasi-isomorphisms. This replicated to all cube (2) and its extension (figure 1).

2. Fundamental Theorems of Modern Research and Engineering Research

A consequence of the before theorem is the following.

Theorem (F. Bulnes) 2. 1. [13]. I). X_{LAB}, is isometric to E_{FISMAT}. II). $\Phi_{\alpha}(t\beta) = X_{LAB}(\Phi_{\alpha})$.
Proof. This can be a simple consequence of the theorem 1. 1. However the details requires use the Parseval's identity and linear isomorphisms.

In the incise I), the Banach spaces X_{LAB}, and E_{FISMAT}, have the same metric of knowledge (in times and advances in research). Topologically both have the same structure as Banach spaces [13, 14]. In the case II), establishes that the obtained results in laboratory must be congruent with the obtained results in the theory. Furthermore, establishes an identity inside the knowledge creation (epistemology of the science) an equivalence inescapable inside the knowledge creation, because much knowledge can be produced with improves of the existing technologies and for other way, the experimental research must be founded in the theory of the knowledge. Both are measurable in duality. This generates certainty and consistence of the research prospective [1, 2].

The field observables are elements of H^{*}, where H^{*}, is the 2-cuadrable measures energy signals space [15].

In modern research, the managing of the time is fundamental. Likewise, are established priorities inside the research classifying to the research projects and lines respect to the time as projects of large, average and short scope. What defines that evolution? The research development. How to establish that this research development is the most optimal? This obeys to the efficient causality principle in the Universe, which affirms that any optimal effect or efficient effect must come from an efficient cause. Then is necessary an optimal design of the cause. Here is where starts. The time is the shortest length between cause and effect.

Proposition (F. Bulnes) 2. 1. The efficient cause inside the research theory comes from a good theory.

Proof. See [1, 5].

For the case of engineering research is opportune consider the following three fundamental aspects of formal engineering theory (FET):

a) **Axiom 2. 1.** The engineering is the creation of the technologies in base of managing of the energy.

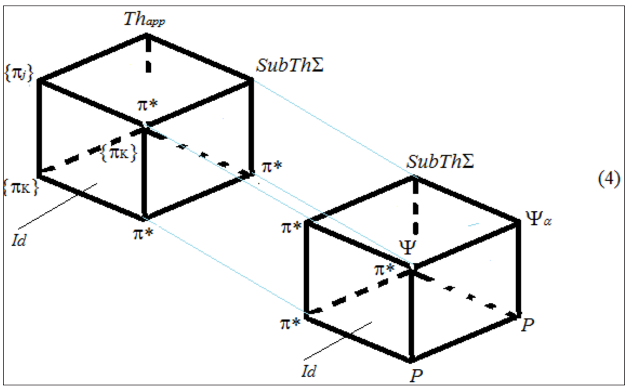
b) **Axiom 2. 2.** The engineering is the final product (technological) of the fundamental sciences: Mathematics and Physics.

c) **Axiom 2. 3.** The engineering is the constant searching of the improvements in process and developments in technology.

Likewise, we consider $EFISMAT$, the category of mathematics and physics knowledge, Φ, the set of mathematics and physics true knowledge Th, the theories category that can generate new knowledge and $\mathfrak{T}\sigma$, the inquiry operator of research. Then who is $\mathfrak{T}\sigma(E_{FISMAT}) \longrightarrow Th\Sigma^{0} \longrightarrow 0$? This is the solution to a theoretical problem derived of the theory $Th\Sigma$. Then in the context of the technological prototypes are considered the following commutative diagram [4, 11, 16],

$$\begin{array}{ccccc} Th_{app} & \rightarrow & SubTh\Sigma & \rightarrow & \Psi_{\alpha} \\ \downarrow & & \downarrow & & \downarrow \\ \{\pi_j\} & \rightarrow & \pi * & \rightarrow & \Psi, \forall\, j \in [\alpha] \\ \downarrow & & \downarrow & & \downarrow \\ \{\pi_K\} & \rightarrow & \pi * & \rightarrow & \mathrm{P} \\ \downarrow & & \downarrow & & \downarrow \\ \{\pi_K\} & \rightarrow & \pi * & \rightarrow & \mathrm{P} \end{array} \qquad (3)$$

which is isomorphic to a cube with the same categories as vertices as follows:



In this case the figure of major weight of (4) is the $Thapp \subset Th\Sigma$ then this is a sub-cube of the cube (1). The schemes are commutative. Here $\Psi\alpha$, is the models algebra of the class α, the set $\{\pi_j\}$, is the category of prototypes which are chosen the $\{\pi_K\}$, to develop through the application of a theory $Thapp \longrightarrow SubTh\Sigma$, the optimal

[2]The category $K(\mathcal{R})$, is of cochain complexes with terms in R.
[3]Quasi-isomorphisms.

prototype $\pi *$. This optimal final prototype is a product P, if can give a solution to a society problem: $P \rightarrow 0$.

Examples and Applications

Example 3. 1. We consider $Thapp$, the category of the applied theory and the homomorphism category $Hom(\phi_{\sigma}(t\gamma),\ t_{\eta})$, which are the technologies generated by the homomorphisms ϕ_{σ}, to t_{η}. Remember that an example of technology is an optimal prototype, $\pi *$, after apply a technologiscism of certain class σ (homomorphism ϕ). Then $Hom_K(\phi_{\sigma}(t\gamma),\ t_{\eta}) \cong \pi *$.

Both categories are isomorphic. Indeed, we consider the face of the cube (4) given by:

$$\begin{array}{ccc} SubTh\Sigma & \leftrightarrow & Th_{app} \\ \downarrow & & \downarrow \\ \pi * & \rightarrow & \pi * \end{array}$$

which is commutative.

Example 3. 2. Then the formal theory of engineering (FET) is the isomorphism (see description of categories in Appendix A):

$$\mathrm{Hom}(Th\mathrm{Mod}\Phi_{\alpha}(E_{FISMAT}), Cn\Phi_{\alpha}) \cong Hom(\phi_{\sigma}(t_{\gamma}),\ t_{\eta}), \quad (5)$$

Since by usual definition, “engineering” is the creation of technologies on bases of the mathematics and physics (axiom 2. 1). However, by the axiom 2. 2, also these technologies are designed and constructed to that function on energy signals. Finally, the perfection in technology is risked with the improvement given by $\phi_{\sigma}(t_{\gamma})$. All this defines the engineering, as knowledge product (axiom 2. 3).

Example 3. 3. Certain research process on rods, is required a special rod with maximum resistance tension with minimal displacements $w(s)$, in its inner. The special rod in our research, could be a technology t_{σ}, on the existing, which we call t_{α}, belonging to the class α, of beams and pillars. First in the level of the theoretical research (considering $EFISMAT \rightarrow \Phi_{\alpha} \rightarrow \mathfrak{T}\Phi_{\alpha} \rightarrow Th\Sigma$) searches what method can be described to this research type as [1, 4] (figure 2).

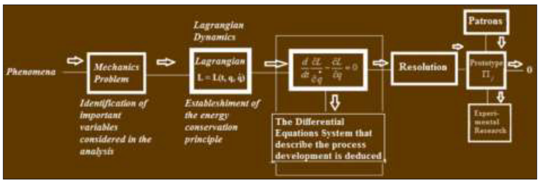


Figure 2: Blocks diagram of research in dynamical of physical systems [1, 5].

After in the prototype level $Thapp \rightarrow SubTh\Sigma$, we have its evaluation given for the value of the following integral (integral execution operator [3]) of execution of the technology

$$Exe(t_{\sigma}) = \int_{\Phi_{\alpha}} \phi_{\gamma}\, \Phi_{\alpha} A(\Phi_{\alpha}(E_{FISMAT}))\mu(\Phi_{\sigma}E), \quad (6)$$

which will derive in an energy integral [17] ($Th\Sigma \rightarrow \{\pi_j\} \rightarrow \pi *$), where the $SubTh\Sigma$, that permits that is the functional analysis and variational calculus:

$$\|w\|^2_{Patron} = \int_0^t GS(t)[w'(2)]^2 dt \leq ES_1 \int_0^t (w^2(t) + w'^2(t))\, dt \ \leq GS_1 \|w(t)\|^2_{1,2}, \quad (7)$$

where G, is the Young´s monulus, $S(t)$, is the area of the cross-section with $S_0 \leq S(t) \leq S_1$, and $w(t)$, is the displacement of the cross-section of the rod at the point t, in the longitudinal direction. We have supposed that at $t = 0$, is fixed, is to say, $w(0) = 0$.

Conclusions

Really the primordial element of a research is a theory represented by the category $Th\Sigma$, in our flow diagram and categorical schemes of commutative type. A fundamental aspect of the category elements of all flow diagram is their homotopy characteristic, which can be seen in the commutative cubes and exposed with more clarity in the correspondence of cubes given in (4) where a cube is homotopic to another. The quasi-isomorphism are determined through inverse morphisms in (1), where finally establish the nature of derived categories of all categories considering the functor of “research” where each component category is of the type $\mathfrak{T}\Phi\alpha$. The research method by fundaments can be a good and precise guide from the mathematics (is to say as rector axis of all research) in any research on any science or knowledge. In a future development on the mathematical research theory will can be developed and designed a tessellation considering the basic commutatively of (1) and its representation as cube (2) and (3), on all a research program. Remember that a research program is a set of many open problems on research, which are related from the basic level until advanced level. Then the research method (1) can be extended or induced to each open problem of a program considering corresponding homotopies (homotopy construction similar to (4)) of the categories as given in the example 3. 2, but carried them to the research unit $Cn\Phi\alpha$.

Appendix A. Basic Definitions and Relations

Def. A. 1. A technologiscism is a neologism that obtains a technology from a technology given. Likewise, $\phi_{\sigma}(t_{\gamma}) = t_{\delta}$.

Def. A. 2. A prototype π, is a technology under research (non-finished product). The theory that generates is a sub-theory or the category $subth\Sigma$. An optimal prototype $\pi *$, is a technology ready to be product P.

Proposition A. 1. [1, 2]. An applied theory is a sub-theory,

therefore $SubTh\Sigma \cong Th_{app}$.

All research unit $Cn\Phi_{\alpha}$, is defined as the place (space) where only exist true propositions. Then a theory based on models of true propositions sets is the category of morphisms given by Hom $(Th\mathrm{Mod}\Phi_{\alpha}(E_{FISMAT}), Cn\Phi_{\alpha})$ is isomorphic to the set of morphisms (creation of technologies) $Hom(\phi_{\sigma}(t_{\gamma}),\ t_{\eta})$.

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