

New Discoveries in Number Theory and its Practical Application

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ABSTRACT

This article shows new properties of positive integers related to the sum of the deviations of the products of dividing the positive number by each of its factors, as well as the positive integer with only three divisors called the perfect square with a square root as a prime number, and the article also shows the building scheme a computer application to find the number of divisors of a number, and to classify the number into prime and composite.

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Introduction

A composite number is that positive integer that can be formed by multiplying two positive numbers less than or equal to it. In addition, it is a positive integer that has at least one divisor other than 1 and itself [1,2]. Every positive integer, prime number, or 1, so the set of constituent numbers is not prime and the number one is not prime because it has only one divisor. For example, the integer 15 is a complex number because it is the result of multiplying two smaller integers of size 3 x 5. The integers 3, 5, and 7 are not composite numbers because they can only be divided by 1 and itself and are called a prime number [1].

The analysis of the number into its factors is from positive integers, without the presence of the remainder as a result of the process of dividing the number by each of these factors, and when all these conditions are met, these numbers are called the divisors of the number [2].

The topic of the current article studies the sum of the deviations of the products of dividing a positive integer by its various divisors. This topic has not been addressed before by scientists and researchers in the field of mathematics. The article also links the number of divisors to any number and gives a new rule for the prime number.

Preliminary

A divisor, or factor, is a number that divides evenly into a larger integer, and the Set up the equation for determining the number of divisors, or factors, in a number [3]. The equation is $d(n)=(a+1)(b+1)(c+1)$, where a, b, c is equal to the number of divisors in the number n , and a, b , and c are the exponents in the prime factorization equation for the number [4].

There are several methods for finding the denominators of a number, including the simple method, which is based on passing

all digits from 1 to the given number and checking that the given number is divisible by those digits, and the pairs method, which is based on when binary denominators are written as pairs that are The product of the two numbers in each pair equals the specified number, so that one number is taken from each pair.

You might have less than three or more than three exponents. The formula simply states to multiply together whatever number of exponents you are working with.

If the prime factorization of a number N is $N = k^a \times m^b \times j^c$, where k, m , and j are different prime numbers, and a, b, c are natural numbers. then the total number of divisors of N is $(a+1)(b+1)(c+1)$ [5].

A prime number is a natural number greater than 1 that is not a product of two smaller natural numbers. A natural number greater than 1 that is not prime is called a composite number. For example, 5 is prime because the only ways of writing it as a product, 1×5 or 5×1 , involve 5 itself. However, 4 is composite because it is a product (2×2) in which both numbers are smaller than 4. Primes are central in number theory because of the fundamental theorem of arithmetic: every natural number greater than 1 is either a prime itself or can be factorized as a product of primes that is unique up to their order [6].

Points to be Noted

- Numbers having even numbers in one' place cannot be a prime number.
- Only 2 is an even prime number; all the rest prime numbers are odd numbers.
- To find whether a larger number is prime or not, add all the digits in a number, if the sum is divisible by 3 it is not a prime number.
- Except 2 and 3, all the other prime numbers can be expressed in the general form as $6n + 1$ or $6n - 1$, where n is the natural number [7].

Properties of Prime Numbers

- Any prime number greater than 3 is written as $6k+1$ or $6k-1$ where k is a natural number.
- Every integer $n > 1$ has a prime divisor.
- If n is a composite (non-prime) number, it has a prime divisor p that is less than or equal to the square root of n .
- If the difference between two prime numbers is 2, then these two numbers are called twin primes. 5 and 7 on one side and 11 and 13 on the other side, they are first twins. (conjecture of twin prime numbers) [8].

Result

The researcher studied and examined many numerical examples in order to reach relationships that link the number system, which constitute the basic building block in mathematics, as the researcher started with the examples listed below.

From below examples

Example :1 30 even *number*

$$\begin{array}{l} 30 \div 30 = 1 \uparrow 1 \\ 30 \div 15 = 2 \uparrow 1 \\ 30 \div 10 = 3 \uparrow 2 \\ 30 \div 6 = 5 \uparrow 3 \\ 30 \div 5 = 6 \uparrow 1 \\ 30 \div 3 = 10 \uparrow 4 \\ 30 \div 2 = 15 \uparrow 5 \\ 30 \div 1 = 30 \uparrow 15 \end{array}$$

conclude

- The sum of the absolute differences between the results of dividing a number by its divisors, respectively. $1+1+2+1+4+5+15=29= N-1$
- Absolute differences without products divided by itself and by 1: $N/2)-2, 30/2)-2=13$

Example :2 18 even *number*

$$\begin{array}{l} 18 \div 18 = 1 \uparrow 1 \text{ Differences } 1+1+3+3+9=17= N-1 \quad (N/2)-2 \\ 18 \div 9 = 2 \uparrow \\ 18 \div 6 = 3 \uparrow 1 \\ 18 \div 3 = 6 \uparrow 3 \\ 18 \div 2 = 9 \uparrow 3 \\ 18 \div 1 = 18 \uparrow 9 \end{array}$$

Absolute differences without products divided by itself and by 1: $N/2)-2$

Example :3 12 even *number*

$$\begin{array}{l} 12 \div 12 = 1 \uparrow 1 \text{ Differences } 1+1+1+2+6=11= N-1 \\ 12 \div 6 = 2 \uparrow \\ 12 \div 4 = 3 \uparrow 1 \\ 12 \div 3 = 4 \uparrow 1 \\ 12 \div 2 = 6 \uparrow 2 \\ 12 \div 1 = 12 \uparrow 6 \end{array}$$

The sum of the absolute differences between the results of dividing a number by its divisors, respectively

Example :4 10 even *number*

$$\begin{array}{l} 10 \div 10 = 1 \uparrow 1 \text{ Differences } 1+3+5 (N/2)-2, 10/2)-2=3 \\ 10 \div 5 = 2 \uparrow 3 \\ 10 \div 2 = 5 \uparrow 5 \\ 10 \div 1 = 10 \uparrow \end{array}$$

Example :5 14 even *number*

$$\begin{array}{l} 14 \div 14 = 1 \uparrow 1 \text{ Differences } 1+5+7=13 \\ 14 \div 7 = 2 \uparrow 5 \\ 14 \div 2 = 7 \uparrow 7 \\ 14 \div 1 = 14 \uparrow \end{array}$$

The sum of the absolute differences between the results of dividing a number by its divisors, respectively

Example: 6 4 even *number* and a perfect square for a prime number

$$\begin{array}{l} 4 \div 4 = 1 \text{ Differences } 1+2=3 \\ 4 \div 2 = 2 \\ 4 \div 1 = 4 \end{array}$$

The sum of the absolute differences between the results of dividing a number by its divisors, respectively

Example: 7 2 even *number*

$$\begin{array}{l} 2 \div 2 = 1 \text{ Differences } 1 \\ 2 \div 1 = 2 \uparrow \end{array}$$

The sum of the absolute differences between the results of dividing a number by its divisors, respectively

Example: 8 25 odd *number* and a perfect square for a prime number

$$\begin{array}{l} 25 \div 25 = 1 \text{ Differences } 4+20=24 \\ 25 \div 5 = 5 \uparrow 20 \\ 25 \div 1 = 25 \uparrow \end{array}$$

The sum of the absolute differences between the results of dividing a number by its divisors, respectively

Example: 9 49 odd *number* and a perfect square for a prime number

$$\begin{array}{l} 49 \div 49 = 1 \text{ Differences } 6 \\ 49 \div 7 = 7 \uparrow \\ 49 \div 1 = 49 \uparrow 42 \end{array}$$

$$6+42=48$$

Example :10 7 odd and prime *number*

$$\begin{array}{l} 7 \div 7 = 1 \uparrow 6 \text{ Differences } 6=6 \quad (N/2)-2 \\ 7 \div 1 = 7 \uparrow \end{array}$$

The sum of the absolute differences between the results of dividing a number by its divisors, respectively

Example :11 9 odd **number** and a perfect square for a prime number
 $9 \div 9 = 1$ 2 Differences $2+6=8$, $(N/2)-2$
 $9 \div 3 = 3$ 6
 $9 \div 1 = 9$ 6

The sum of the absolute differences between the results of dividing a number by its divisors, respectively

Example :13 21 odd **number** Differences $2+4+14=20$
 $21 \div 21 = 1$ 2
 $21 \div 7 = 3$ 4
 $21 \div 3 = 7$ 14
 $21 \div 1 = 21$ 14

The sum of the absolute differences between the results of dividing a number by its divisors, respectively

33 odd **number** Differences $2+8+22=32$
 $33 \div 33 = 1$ 2
 $33 \div 11 = 3$ 8
 $33 \div 3 = 11$ 22
 $33 \div 1 = 33$ 22

The sum of the absolute differences between the results of dividing a number by its divisors, respectively

Example :14 63 odd **number** Differences $2+4+2+12+42=62$
 $63 \div 63 = 1$ 2
 $63 \div 21 = 3$ 4
 $63 \div 9 = 7$ 2
 $63 \div 7 = 9$ 12
 $63 \div 3 = 21$ 42
 $63 \div 1 = 63$ 42

The sum of the absolute differences between the results of dividing a number by its divisors, respectively

Example :15 85 odd **number** Differences $4+12+68=84$
 $85 \div 85 = 1$ 4
 $85 \div 17 = 5$ 12
 $85 \div 5 = 17$ 58
 $85 \div 1 = 85$ 68

The sum of the absolute differences between the results of dividing a number by its divisors, respectively

Example :16 1
 $1 \div 1 = 1$ odd number Differences 0

Example :17 0
 $0 \div 1 = 0$ Differences 0
 $0 \div 2 = 0$
 $0 \div 3 = 0$

From the above examples we conclude this rule
 The sum of the absolute difference between the real and non-real divisors of a number is -1
 The sum of the absolute difference between the denominators of a real number is (the number divided by 2)-2
 Any number whose real and non-real divisors sum to 3 is a perfect square of a prime number

The sum of the absolute difference between the real and non-real divisors of a number is -1

Theorem 1
 The sum of the absolute differences between the results of dividing a positive number by its divisors, respectively is equal to that number - 1

.e., If N an integer positive number, a_i is its divisor, n the number of a divisor of the number N, then $\sum_{i=1}^n (a_i - a_{i-1}) = N - 1$

lemma
 The sum of the absolute difference between the results of dividing a positive integer by a divisor without 1 and N is equal to this number divided by the least divisor of the number - the least divisor of the number

i.e. If N a positive integer, and w is the least its divisor then

$$\sum_{i=1}^n (a_i - a_{i-1}) = \frac{N}{w} - w$$

$$\sum_{i \neq k}^n (a_i - a_k) = \frac{N}{3} - 3, \text{ if } N \text{ odd number}$$

If A is a positive integer number that have $1, a_2, \dots$ factors
 $A/a_1 = \text{Type equation here.}$

7
 $72 = 49$
 49 has one real factor only its 7
 Its right
 A perfect square with 3 prime factors has a square root of a prime number

Theorem 2
 If a is a positive integer number, and it has only three divisors, then a is a square complete number, and the square root is a prime number.

i.e., if a is positive integer number, a has 3 divisors, then $a = (m)^2$, m is prime number
 Proof theorem2
 a is a positive integer number,

a it has only three divisors 1, a, m
 $1 \times a = a$

$$m \times n = a \Rightarrow n = m$$

$$m^2 = a \Rightarrow a \text{ is square complete number}$$

$$\sqrt{a} = m \text{ is prime number}$$

Practical Applications of the Theories in this Article
 Finding a computer application in order to determine the number of divisors of the number so that the dividend is a positive integer and the result of the division is an integer, and thus the number that has only two divisors is a prime number, and the number that

has three divisors is a composite number and it is also a perfect square (for a prime number) That is, the square root of a prime number. Through this application, we can know the number of denominators of the number, as well as knowing the prime number from the non-prime

How to Make a Computer Application

First: Enter the number (positive integer) into the application

Second: The application automatically divides the number by all numbers that are less than or equal to half of the number entered into the application.

Third: If there is any number that divides the number entered, the number will write down these numbers starting with the number 1

Fourth: These divisors are counted and 1 is added to them to be the sum of the divisors of this number, and they appear on the screen or the calculator through a button whose symbol is $f(n)$.

Fifth: Classify the number through a special button, and its symbol is $c(n)$, and the number is classified into: prime number, a number with 3 divisors (a perfect square of a prime), a number with more than 3 divisors

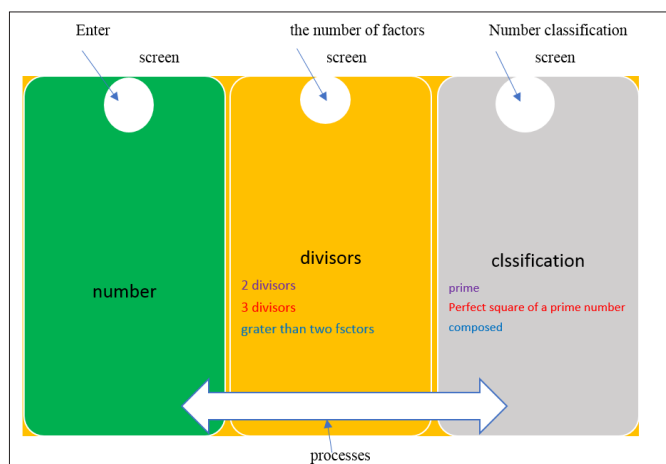


Figure 1: Shows the Computer Application and its Working Mechanism

- Fraleigh, John B. (1976), A First Course In Abstract Algebra (2nd ed.), Reading: Addison-Wesley, ISBN 0-201-01984-1
- Herstein, I. N. (1964), Topics In Algebra, Waltham: Blaisdell Publishing Company, ISBN 978-1114541016
- Long, Calvin T. (1972), Elementary Introduction to Number Theory (2nd ed.), Lexington: D. C. Heath and Company, LCCN 77-171950
- McCoy, Neal H. (1968), Introduction To Modern Algebra, Revised Edition, Boston: Allyn and Bacon, LCCN 68-15225
- Pettofrezzo, Anthony J.; Byrkit, Donald R. (1970), Elements of Number Theory, Englewood Cliffs: Prentice Hall, LCCN 77-81766

Recommendation

The article presented new rules added to the theory of numbers, which benefit scientists and researchers in the field of mathematics and various sciences, and we offer researchers and scientists new rules that they can build on and employ by those interested in programming and new technology, A scheme that programmers can apply practically so that students and scholars can benefit from this application in their studies and applications.

References

1. Carothers (2000) says: "N is the set of natural numbers (positive integers)" (p. 3).
2. Mac Lane & Birkhoff (1999) include zero in the natural numbers: "Intuitively, the set $N = \{0, 1, 2, \dots\}$ of all "natural numbers" may be described as follows: N contains an "initial" number 0; ...". They follow that with their version of the Peano Postulates. (p. 15).
3. <https://www.mathsisfun.com/definitions/divisor-of-an-integer-.htm>.
4. http://mathschallenge.net/library/number/number_of_divisors.
5. https://www.hsoub.com/algorithms/al_divisors <https://www.com/Mathstoon> Divisors of numbers: Definition, Properties, Formulas, Table.
6. https://en.wikipedia.org/wiki/Prime_number.
7. [https://www.vedantu.com/maths/How to Find Prime Numbers?](https://www.vedantu.com/maths/How_to_Find_Prime_Numbers?).
8. <https://ar.wikipedia.org/wiki/>.

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