ISSN: 3033-3652

Journal of Physical Mathematics & its Applications

SCIENTIFIC Research and Community

Review Article

Open Access

Newtonian Mechanics Extension (to Arbitrary Speeds) and its Relation to Special Relativity in Complex Model

Jerzy K Filus

Department of Mathematics and Computer Science, Oakton Community College, Des Plaines, IL 60016, USA

ABSTRACT

Complex Euclidean C^3 model for "para-space", as an alternative to the real Minkowski's M^4 space-time model for special relativity (SR) was introduced. As it turned out, in the SR theory of the complex model, velocities can be defined with no use of time and so time (no more the primitive notion!) can be defined within the theory of 3-D complex model.

The complex C^3 model, initially thought of as the model for special relativity (SR), turned out to possess deep unifying properties as being able to model both SR and classical Newtonian mechanics. The latter classic theory of the C^3 model can also be extended to arbitrary high, but finite, speeds while the "Galilean speed of light" turns out to be infinite. The latter property clearly explains the universality of speed of light phenomenon. This became possible since all the SR phenomena one recovers from the unobserved complex Newtonian simply by taking real parts from underlying complex (para)physical quantities. This unifying property of the theory of the complex model, hypothetically, can be extended also to quantum mechanics so that all the three mechanics can, possibly, be unified as one theory of the complex C^3 para-space model. Besides the unifying properties, use of this complex model dramatically simplifies the underlying physical theories especially (if this hypothesis is true) the quantum mechanics. The related to this complex approach ontological problems of existence of the unobserved physical objects and phenomena in $C^3 – R^3$ were addressed in the Appendix.

***Corresponding author**
Jerzy K Filus, Department of Mathematics and Computer Science, Oakton Community College, Des Plaines, IL 60016, USA.

Received: October 29, 2024; **Accepted:** November 05, 2024; **Published:** December 04, 2024

Introduction
This work complements and enriches my previous paper as well as [1,2]. There are two main reasons that inclined me for writing the second (improved) version of that paper. First is the need to improve the previous text at some points in order to deepen or just change a bit my previous ideas. Secondly, in recent time, I have realized that as for the constructed complex version of the Lorentz transformation the time transformation needs to be amended by adding to the primary formula some lacking essential coefficient. The full justification of that new (true) version of the complex Lorentz transformation required creation of the present paper. In the following text I begin with the old ("intermediate") version of the transformation.

I introduce and justify the new. Also, justification of (the same) mass transformation in section 6 is new.

The complex model construction we begin with a simple and basic mathematical trick that yields a very efficient and simple model for a wide range of physics including Newtonian mechanics, special relativity, QM and possibly more [3]. This trick and the following it mathematical model for a wide range of physics seems to have been overlooked for more than a hundred years. The reason for that is unknown but it seems that the tremendous lack of trust in using complex numbers in a little more literal way may be the main cause of it. As a result, we face still growing with the rocket speed complexity of mathematical apparatus with the corresponding physical context often being lost. The simplification of mathematical formalism in order to enrich the underlying physical analysis seems to be one of the most vital tasks of todays in theoretical physics.

A remedy that can help with this task (at least to a significant extend) could be the proposed and analyzed in this work model. To construct it the just mentioned simple mathematical trick involving complex numbers on an elementary level had to be applied. What is the most essential, however, is a transition from some physical concepts based on real numbers to corresponding physical (or, possibly, "*paraphysical*") concepts based on the model involving complex numbers. As the first step, the classical real Lorentz transformation in the Minkowski M^4 model was extended to its complex counterpart in the constructed complex space time C^4.

The fundamental **trick** relies on two steps. First, the common in relativity (especially in Lorentz transformation) expression $\sqrt{(1 – u^2 /c^2)}$ replace by the trigonometric expression $\cos\theta$ together with $u / c = \sin\theta$, for the same value θ. Second, by an "educated guess", add to the coefficient '$\cos\theta$' its usual imaginary counterpart '$i \sin\theta$' ($i^2 = -1$).

Altogether, the factor $\sqrt{(1 – u^2 /c^2)}$ was replaced by the rotation $\exp[i \theta]$ in the so constructed complex planes which extend the real straight lines (x-axis and t-axis in M^4, for example).

As the result the relativistic motion along the x-axis was shown to be equivalent to the Newtonian (!) within interior of the so obtained complex domain. That Newtonian motion is extended to arbitrarily high [but unobserved] speeds (!) while the corresponding 'complex speeds of light' (their absolute values equal to C = c /cosθ, as θ varies) in the parallel directions (same θ) in complex plane are still higher than the speeds of the physical bodies (as it is shown) by the same common value c.

In the new complex model (with the three axes from R^3 extended by the class of rotations exp[iθ_j] (j = 1, 2, 3) to complex planes from C^3 and with time epochs also being points of other complex plane) one (almost) can recover the usual classical mechanics with mass remaining the rest mass m_0 regardless of speed magnitudes and without the mysterious contractions of the distances (lengths).

On the other hand, the relativistic SR theory can be easily recovered upon the projection of Galilean motion from C^3 to R^3 (by taking the real parts of complex Newtonian quantities) [3]. Thus, the so projected unbounded (!) Galilean speeds again become the relativistic speeds all bounded by (relativistic) speed of light c. In order to preserve the Newtonian (rest) mass in complex directions together with preserving magnitudes of the relativistic masses in the real direction we introduced (or discovered?) the "imaginary mass" i.e., the mass measured in imaginary units "[i kg]" (in my notation [*kg]) whose square is negative (- kg^2). This notion differs from the similar notions applied in quantum field theory at least by the fact that (possibly first time) it was defined and applied in the classical macro context [4-6].

The "trick" relying on replacing in the Lorentz transformation formula the quantity $\sqrt{(1 - u^2/c^2)}$ by the complex plane rotation exp [i θ] turned out to be extremely fruitful. First of all, the so created theory, [which at least contains both the (extended to all speeds) classical mechanics, special relativity and very likely also the QM] is quite unexpectedly **consistent** and simple.

Besides, all properties and theorems of special relativity are preserved upon the C4 → R4 projection. On the other hand, many of "mysteries" behaviors well known in SR (such as, for example, the universality of speed of light or Lorentz contraction) has a natural explanation in the wider theory which at least contains both classical and SR theories.

Complex models for space and time are known in literature (although are not very common. Nevertheless, the ways they are introduced are different (**not** by the rotations of real axis into the, so constructed, complex planes) than the here presented [2-10].

According to my best knowledge, there is no complex model, as far as up to the now constructed, in which both the Newtonian (extended) and relativistic theories are satisfied.

On the other hand, some authors stress the fact that within the interior of "physical complex space" outside of its real part, say R^4, some *real physical phenomena* have likely been taking place, see for example [8].

The problem that arises is lack of direct empirical evidence of such phenomena's existence.

However, some authors find (as also I do) an alternative for a direct empiric material in an inner consistency of hypothetical physical phenomena with the phenomena observed by physical instruments. By the way, the situation is not much different to the situation the physicists face in theories of elementary particles that we never directly observe.

Thus, in my opinion, there is no reason to deny existence of entities not given by a direct observation (see, Appendix) as some (but not all) methodological theories or approaches, based on radical empiricism, claim.

Direct physical observation by physical instruments is not the only option in bringing evidence of physical existence. Other criterion may be, the given by a mathematical model, logical **consistency** in existence of hypothetical objects and phenomena with the phenomena empirically found by regular physical experiments that obey well-grounded theories such as SR.

Some methodological and ontological analysis that deals with this kind of problems is included in the Appendix.

The starting point of the whole theory i.e., the complex extension of real Lorentz transformation by use of the, above mentioned, 'mathematical trick' is presented in section 2 and more intuition on that, where the complex plane of the motion is illustrated by Figure 1, fulfills section 3.

One of the main reasons I present this paper, while a significant amount of the results were already published in some inadequacy (that recently I realized) in the previous version when the complex time transformation was established [1]. In this time transformation was proportional (by the coefficient c) to the length transformation so actually both the transformations were essentially identical [1]. Analyzing closer time phenomena such as, by example, 'twin paradox' and other I found that the complex time transformation differs from the transformation of space. Instead of simple rotation (as in the space case) we have a bit more sophisticated "rotation" of time where not only the angle θ changes but also the radius (the time at rest) contracts by the coefficient 'cosθ'.

This fact changes the original theory as presented in [1,2]. On the other hand, based on my personal experience, I have to say, the final theory as proposed in this paper, would not, probably, be well understood by many if it wouldn't be proceeded by that (simpler) "intermediate theory" as presented in [1].

That is why, even in this paper, I refer to the "old transformation" (3) as well as I discuss it as the "first approach" in section 4 together with ('false') Figure 3 as "pedagogic means" for preparation to the finally correct complex time transformation (13) ("second approach") in section 5 [see also Figure 3*]. For that, also in section 5, I analyze first, Galilean speeds, however from some different perspectives then in [1].

It needs now to be noticed that the regular real Lorentz transformation (1a) or (2) one can recover from the new complex transformation (13), now by taking real parts from the first three rows (space transformation) and the absolute value from the last row (the time transformation). The SR theory is still preserved while the corresponding Newtonian mechanics is better grounded in the new general C3 – theory.

In section 6, similarly like in I analyze mass of physical body when it moves in the complex (natural) direction [as determined by the corresponding to its speed angle θ] [1]. According to my claim on the Newtonian character of any motion in the natural direction the mass should have the invariant (in speeds) value m_0. The justification of this fact is much more complete and a bit different than that in [1]. In order to avoid a little artificial, from physics viewpoint, mathematical concept of the "hyperbolic complex numbers" and the corresponding "hyperbolic absolute value" as introduced in instead, in section 6 I introduced a notion of "imaginary physical units" that is, by an assumption, different than the notion of an 'imaginary number'[1]. These two notions, regardless of formal similarity have different roots and natures. The imaginary physical units (in particular, the imaginary units of mass) are assumed to be nonmathematical objects expressed in a language that do not belong to mathematics while belonging to, say, wider language of "(para)physics". The latter, together with some geometric considerations allowed to define "complex Newtonian mass" (being the combined 'math – phys.' object) whose (this time, regular) absolute value equals to, always the same, rest mass m_0. The imaginary mass as defined by the imaginary mass units ([*kg] so that $[*kg]^2 = - kg^2$)

for macroscale objects is, according to my best knowledge, a new concept in literature, different from similar concepts introduced, especially, in quantum field theory (see, for example, the Higgs fields) [9-11].

In section 7 some rough formulation of Newton differential equations for motion in the complex model and its relation to their relativistic version is given.

The paper ends with an Appendix in which I analyze an 'existence ontological problem' for the introduced objects situated in C^3 outside of its real part R^3 as well as the physical (or rather "para-physical") 'meaning' of the entity modeled by C^3. The Appendix, unlike the preceding sections 2 – 7, have a philosophical character rather than a physical and can be omitted by purely physically oriented readers.

As for the general concept of *existence* I assumed that "something does exist if it is significantly related (having an "impact" on) to objects previously known to exist".

This methodology suggests the need of axiomatic approach to ontological problems. Upon this choice of the existence definition, as well as, supported by the known physics, existence of regular objects in R^3, I concluded some existence of the objects in $C^3 – R^3$. After that, I stated the problem of *nature* of that existence that actually remains open.

Since the objects need not to be considered as physical (first of all, are essentially unsensual and not directly detectable by physical instruments) their existence is considered as "para-physical" as strongly associated with the regular physical objects. The possible closer interpretation of the *para-physicality* may be either mental or spiritual or formal (mathematical) or just "extended physical". One possible version (interpretation), associated with the spiritual and mental, is theological that identifies the whole such reality with transcendental God. As almost all ontological problems this problem of interpretation is open and not univocal.

The Lorentz Transformation as Departure Point from the Real Minkowski M4 to the Complex C4 Model Transition

Let me recall remarks on the 'complex Lorentz transformations' in reference of my previous papers [1,2].

As it is commonly known, the original real transformation is defined in Minkowski M^4 space-time typically as follows:

$$x - ut = x' (1 – u^2 / c^2)^{1/2}$$

$$y = y'$$

$$z = z'$$

$$t – ux / c^2 = t' (1 – u^2 / c^2)^{1/2} . \quad (1a)$$

In the above, one considers the fast motion along the x-axis of M^4 with a constant speed u.
At first, the attention was restricted to the first line of (1a), and compared with the corresponding part of the **Galileo** transformation:

$$x - ut = x'. \quad (1b)$$

The two transformations of space only differ by the familiar factor

$$\mathbf{(1 – u^2 / c^2)^{1/2}} .$$

Then it was realized that simple pure mathematical observation gives the "trigonometric representation" of this factor, together with another, also persistently occurring in special relativity (SR) theory, quantity ' **u / c** '.

As it was realized, both the quantities $(1 – u^2 / c^2)^{1/2}$ and u / c may be considered as cosine and sine, respectively, of some common "angle", say, θ.

So, I applied the following notation:

u / c = sin θ, and $\mathbf{(1 – u^2 / c^2)^{1/2} = \cos\theta}$

and then rewritted (1a) to the form:

$$\mathbf{x - ut = x' \cos\theta}$$

$$y = y'$$

$$z = z'$$

$$\mathbf{t – ux / c2 = t' \cos\theta}. \quad (2)$$

The question that arises at this point is a possible 'physical interpretation' of that "angle" **θ** that first might appear to be "*mysterious*" or just "meaningless".

The following "thought experiment" that was performed was initiated by the question:

"What would [physically] happen if we extended the factor 'cos θ 'in (2) by adding its (mathematically very natural) "*imaginary counterpart*": ' i sin θ ', where $i^2 = -1$ " ?.

As for the beginning, the only motivation for that, purely mathematical "action", was a mathematical intuition.

But by pursuing that way further one observes that, the above "*educated guess*" quickly implies several (unobserved directly in reality) "*physical properties*" that are "felt" to be strongly desirable for a better "rationality" and simplicity of a theory of the "*physical motion*".

Notice that, after adding the imaginary term 'i sinθ' to the coefficient 'cosθ', the Lorentz formula (2) results in an extremely useful form of the complex transformation:

$$\mathbf{x - ut = x' \exp[\, i\, \theta\,]}$$

$$\mathbf{y = y'}$$

$$\mathbf{z = z'}$$

$$\mathbf{t - ux / c^2 = t' \exp[\, i\, \theta\,],} \qquad (3)$$

Remark: As already mentioned in section 1, version (3) (old or "intermediate version") of complex Lorentz transformation, as considered in [3], turned out not yet to be the final one. According to further considerations in this paper (section 5, the "new version") the last row in (3) should rather be replaced by the following:

$$\mathbf{t - ux / c^2 = (t' \cos\theta) \exp[\, i\, \theta\,],}$$

while the first three rows are to be preserved (see, formula (13)). Thus, in order the classical mechanics (the Galilean speeds) along the radius r in the complex plane works, the complex time t must (according to "new version") transform differently than the (complex) length x.

Now, until section 5, I will shortly recall my previous ("old") comments on transformation (3) to make easier the transition from (3) to the true ("new") version (13) (In my opinion (13), without the "intermediate stage" (3), may be seen unintuitive and complicated. Besides, space transformation in both (3) and (13) are "roughly" the same.) [1].

Thus, according to (3), the quantities x and t became complex numbers. Their real parts Re x, Re t may be identified with the former real quantities x, t as present in (1a) and (2). The latter represent the result of measurements obtained by the rest observer located at position (0,0) , i.e., at the origin of the so created complex plane (see Figure 1). Unlike x, t in (3) the quantities x', t', measured by the "moving observer" (who is situated on the "back of a rocket"), are real and for the absolute values we obtain:

$$|\, x'\, | = |\, x - ut\, | \qquad (4)$$

and

$$|\, t'\, | = |\, t - ux / c^2\, |, \qquad (5)$$

respectively, due to the common fact that $|\exp[\, i\, \theta\,]\,| = 1$ for each value of θ.

The symbol | . | denotes the absolute value of a real and a complex number.

The rest observer does not see, by his senses nor instruments, objects expressed by complex numbers x, t but we will assume he can "see" these '*intelligible realms*' (together with all the interior of underlying complex planes) "mentally", for example by means of the accompanying mathematical model.

In literature the 'imaginary (or part of) time' often is formally associated with the "motion" at an 'imaginary (space) distance' (see [7]).

The notion of 'complex time' is relatively widely applied in the literature, especially in quantum field theory. It is often considered in association with the constructions of four-dimensional complex manifolds [12].

These manifolds often are considered to be complex extensions of the real M^4 –Minkowski spacetime, so the corresponding "metrics" (actually, as given by the semi-Riemannian scalar product "metrics" are not metrics at all in the regular topological sense) remains non-Euclidean and for this reason such complex spacetimes are essentially different (in topological structure) from the common complex Euclidean space C^4 that we intend to apply in association with (3) [7,9].

Realize, that (by the definition of the angle θ) for the speed u we have:

$$u = c \sin\theta. \qquad (6)$$

Here, notice that defining (by (6)) speed geometrically only, by means of the angle θ, I avoided use of the concept of time. Moreover, the present in formula (6) speed of light c may also be considered as time free geometric concept equivalent to the angle $\theta = \pm \pi/2$ that is to the motion parallel to the imaginary axis.

[Thus, any speed can be measured in radians or degrees.]

As it was argued in the above realization and other yield to the claim that notion of time is not a 'primitive notion' and can be deduced from geometric properties of the spatial part C^3 of the complex space time C^4 [1]. The latter and some additional analysis of the complex Lorentz transformation (3) resulted in reducing the complex space-time C^4 model to the "para-space" C^3 model in which theory 'time' can be defined (and so C^4 or, say, C3,1) can, eventually, be recovered from the C^3) in pure geometric manner [1].

In the next two sections we analyze (according to both versions) some simplified illustrations of motion within the considered complex plane that extends the physically observed motion along the real straight line [i.e., along the originally given x-axis of R^3].

Similar description concerning the "flow" of complex time in complex time plane, follows.

The graphs and most of the comments are taken from [3], so until section 5 they represent

the slightly modified, old version of the C^3 and C^1 (considered separately) theory.

The Complex Phenomena' Illustrations

We analyze the (complex versus real) motion as illustrated in Figure 1:

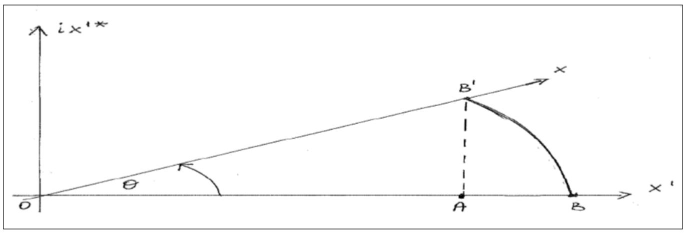


Figure 1

In Figure 1, the 'rest observer' and the 'moving observer' (both placed either in or "next to" [i.e., at the zero distance from] the back of rocket) are assumed to be situated at the origin O of the x' + ix'* complex plane (of complex positions) at the moments t = t' = 0.

Compare two different models for the same rocket motion.

First, for a while, consider (as in regular SR) only the real axis (call it, also, x'-axis) ignoring rest of the complex plane.

In this case, the rocket, as seen by the rocket observer, is "spread out" between the points 0
and B. "At the same time" for rest observer it is spread between 0 and A, with A < B, due to the regular Lorentz contraction.

Return to the whole complex plane (the second model). Now, the "true trajectory" of any object which moves with the speed u = c sinθ differs from the real direction by the angle θ so that the rocket moves along the (radial) direction x.

In this case, both "**complex observers**" [the ones, that can "mentally see" the whole complex plane of the motion, say, hypothetically or within the mathematical model] this in the rocket and this at rest, see front of the rocket in the same complex position B'. This fact is in agreement with Newtonian mechanics (in its complex version), where there is no difference between the two observations of the position once the observers are situated at the same position.

However, once it comes to the real results interpretation (the corresponding observations in the real space) of the complex position B', the real position for rest observer is at A = Re B' (the real part of complex B'), while for rocket observer the interpretation of the position is B = |B'|. In the first case, the rest observer, once within the real space, sees the projection (the "shadow") A of the "true position" B' [only by "ignoring" its imaginary part |AB'|]. Unlike, the rocket observer sees the real position at B "ignoring" its ("true") rotation by the angle θ. In a quite good sense, one can say, that both the relativistic (real) positions A and B have the same Newtonian (complex) "source" B'. Splitting one complex Newtonian value B' into two relativistic measurements A, B (one being the Lorentz contraction of other) results in transition from complex space to its real part.

By the way, looking at "earth" (the x' – axis) from, say, B' position, rocket observer sees it as a 'moving object' and as such it is subjected to the Lorentz contraction. This Lorentz contraction causes the whole metrics of the "moving earth" to be contracted with the coefficient |0A| / |0B| = cosθ. This Lorentz phenomena may also be understood, geometrically, as the result of projection of the "natural metrics line" OB' into the "relativistic line" OA.

In the mathematical (complex) model we will consider a body's complex position on the line 0 B' as "**natural** position" and the 0 B' line direction as "*natural direction*".

The adjective "*natural*" for the word "*direction*" will be given regardless the fact that this direction is out of direct physical (and sensual) observation.

As for the body's real direction OA, I would propose to call it the "observable direction".

Needless to say, the knowledge of the 'rocket's' observable parts OA and OB (each given only to one of the two observers) uniquely determines *the natural position* OB'. For that, notice that the only needed angle θ one always can obtain from: cos θ = |OA| / |OB|. Therefore, in this sense, complex positions are at least "*mentally observable*" (understood, computable, possibly automatically illustrated on the screen of an accompanying computer).

Now, consider the following 1-dimensional figure:

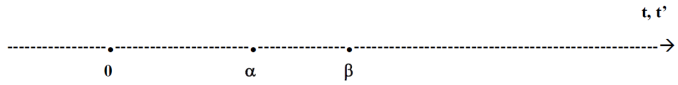


Figure 2

Figure 2 geometrically illustrates Lorentz' real time transformation t' → t [now, both t and t' are real] , whose analytic version is given as the fourth line of transformation (1a) or equivalently (2).

So, at the moment, we restrict the analysis to the real line only (the t-axis and t'-axis are distinct, parallel with the same origin).

Referring to Figure 1, suppose both observers are located at the origin 0, which corresponds to the position on the back of the rocket.

Realize, that the "information" (observation) about the rocket's front will be available after the times $\alpha = t_A = |OA| / c$ for the rest observer and $\alpha = t'_B = |OB| / c$ for the rocket observer.

More generally, these relations are satisfied by any position x and any corresponding x'. The fact that $\alpha / \beta = t_A / t'_B = x / x' = \cos\theta$ is due to the well-known **time dilation** phenomenon.

This phenomenon and, more generally, 'time transformation' has an explanation if we consider the complex version of time as modeled by the complex plane. The (true) results are not straightforward and I will present them in two different stages. First stage (section 4) yields the results (together with the complex Lorentz transformation (3)) already presented in [3] (the old version). The second stage (in section 5) contains a new approach and new results as for the time transformation. As mentioned, it turned out to be necessary to modify the complex Lorentz transformation (3) to its new version (13). However, since the first version comes so naturally in the investigation process it may be treated as a natural intermediate 'stage of thought' (not the reality) between the original real Lorentz transformation (1a) or (2) and the result (13) that I believe is final. As it will be seen, both complex versions are reducible to the same original real Lorentz (1a).

First, let me shortly describe the old "naïve" version equivalent to transformation (3) as presented in [1].

Complex Time Transformations; The Intermediate Version

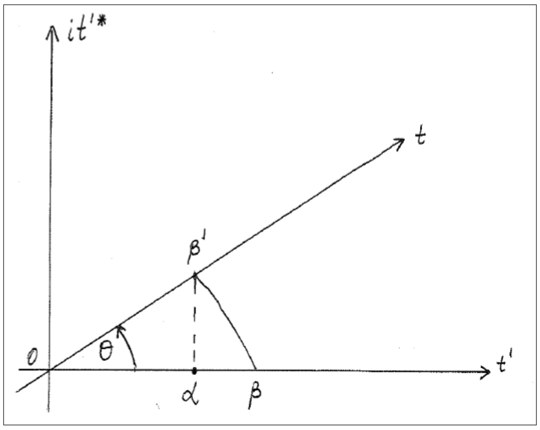


Figure 3

For better clarity consider the basic "rotation of the (complex) time t" to be performed about the origin 0 (of the 'time complex plane') instead of about the arbitrary point $t0' = ux/c^2$.
Analytically this corresponds to the assumption that x = x' = 0 in (3).

Now we transform the formulas (1a) and (2) into (3), and set the originally real (parallel) lines x and x' as well as t and t' into the complex plane (Figures 1 and 3 respectively). Doing so, one recovers the ("absolute" according to the old version of the transformation) time invariance [in the old (intermediate) version], since the line t (initially parallel to t') is just rotated by the angle θ.

As a result of that rotation the real time point β (corresponding to the real space position at point B) turns into complex time β' (corresponding to the complex position B').

The Newtonian 'time invariance' would be, in such a case, recovered upon taking the absolute value of the "complex time" β' (so |β'| would, possibly, be considered as the "**Newtonian absolute time**"). This can be expressed (in the same way as for the space) by the relation: |**O**β| = |Oβ'| (Recall, that's only according to the intermediate version.).

The dilated [relativistic] time |Oα| (the real part of the complex time), as observed by physical instruments of the rest observer, turns out to be the **projection** of the complex time point β' onto the real line of time t'.

The above intermediate ("naïve") version of the time transformation could be seen as an "attractive" for its simplicity and by an emerged temptation of recovering the Newtonian "absolute time". However, dipper considerations incline to slightly different interpretation of the complex transformation of time that results in a new version of the Lorentz transformation (13). From now on (13) we will call the "**modified complex Lorentz transformation**", which seems to be the (final) "true transformation".

On New Time Transformations

Galilean Speeds: As it soon will be seen, time transformation strongly depends on a specific 'speeds transformations', the subject rather unknown in literature in the form below presented. We then start time analysis with analyzing speeds as in the complex model speeds may be defined purely geometrically without using any prior time measure.

Meanwhile, since I start here the new (final) version of complex Lorentz transformation' description, the notation will slightly be changed.

Namely, from now on the complex variables x, t in Figures 1 and 3 as well as in transformation (3), will be denoted by x_c and t_c, respectively.

Recall, there is one to one relationship (6) between any speed u and the (purely geometric notion) angle θ. Speed u is then defined as the product 'c sinθ', where the constant c is the speed of light in vacuum that geometrically, in the complex plane, may be identified with the notion of "orthogonality" or with the angle π/2. In other words, "speed θ" tells "*how far* [possibly by a "measure": '1 – sinθ'] is a given speed u from the orthogonality". I will not go much further into this geometric interpretation of the c, noticing that it only makes sense when the plane is complex and that Imaginarity (or the imaginary direction) seems to exhibit its nature as a "source" of motion (eventually 'energy') when *physical* space is modeled by the C^3.

Now, that seems to be natural that having at our disposal notions of distances and speeds, especially the speed of light (at zero "distance" [= 1 – sin π/2] from the orthogonality), one can define time so the time is not a primary notion in this complex model framework.

At the moment, let us concentrate on speeds of physical bodies that, according to the earlier considerations, can move in the interior of the complex space. For example, consider a (classic) particle, that moves along the line 0B' (Figure 1), and its position is to be identified with the point B'. Notice, its trajectory determines uniquely (relativistic) speed u = c sin θ.

It is natural, in this case, to consider the "speed" of the particle B' as having the radial direction 0B' while u as this direction's horizontal (real) component. This implies an existence of a vertical (imaginary) component of the radial speed [in the (separate) plane of speeds]. It is the speed along an imaginary distance, say, AB'. That suggests that the radial speed's magnitude, say, U is bigger, in absolute value, than u.

Otherwise (assuming that speed in radial direction and its horizontal and vertical "coordinates" obey the usual rules of vector calculus) if U = u then the horizontal coordinate of the radial speed had to be shorter than u, but this is not the case as there are no two different speeds of the same particle in the real direction.

Therefore, it is necessary to consider U > u, whenever none of the two speeds is zero. We claim that

$$U = u \sec \theta \qquad (7)$$

where $-\pi/2 < \theta < \pi/2$.

In order to show (7), consider as an illustration Figure 1. Suppose, this time, that vector 0A represents the real speed u, while the vector 0B' is to be considered the corresponding 'complex speed' say, U_c corresponding to the complex number B'. Denote U = ±| U_c | (U is real and it takes on negative values whenever u is negative). Since the points 0, A, B' form the right triangle we obtain |0B'| / |0A| = U / u = secθ that proves (7).

Substituting in (7) $u = c \sin\theta$, one obtains other key formula:

$$U = c \tan \theta \quad (8)$$

From the latter it follows that the "radial speeds" U (the signed absolute values of the complex speeds U_c) are **unbounded** since, as $\theta \rightarrow \pi/2$ (and so $u \rightarrow c$), we have that $U \rightarrow \infty$.
Fortunately, they still are smaller than the corresponding to each (separately) "speed of light".

Such "speeds of light", denote them by $C = \pm |C_c|$, are speeds the light travel in radial direction θ, after being sent from, say, a rocket that is moving in that direction (i. e., with speed U).
Using similar argumentation as in justifying formula (7) one obtains any such speed as

$$C = c \sec\theta \quad (9)$$

In this case, the **Einstein' universality of speed of light** may be considered as preserved in a nice Pythagorean manner after observing that, for every θ, we have:

$$U^2 + c^2 = C^2.$$

The latter follows from formulae (8), (9).

More on this and other specific forms of 'universality of speed c of light' can be found in [3].

Needless to say, these real speeds C are the ordinary signed absolute values of the "complex

speed of light" C_c and the following formula is satisfied:

$$C_c = Ce^{i\theta} = c + i\, c \tan\theta. \quad (10)$$

As it is seen from above, its real part is the ordinary relativistic speed of light (in the real direction) c, while its imaginary part is the speed U of medium (here the rocket) as given by (8).

This stands as another argument for the preservation of universality of speed of light c in this simple additive form (the "difference" of the speeds is always c, regardless of θ), see [1].

To the transformation of the "generalized" speed of light (10) we add the obvious transformation of all other speeds:

$$U_c = Ue^{i\theta} = u + i\, u \tan\theta \quad (11)$$

As for the "*actual* (*pure*) *speed of light*" C_{abs}, it should, by nature, correspond to the angle $\theta = \pi/2$.

Notice that, in accordance with (9) and (10),

$$Cabs = \lim_{\theta \rightarrow \pi/2} | C_c | = \infty,$$

while $\lim_{\theta \rightarrow \pi/2} \mathrm{Re}\, C_c = c$ as the limit of the constant function c of θ.

(Here, realize, we always have $C_c = C_c(\theta)$ as given by (10).)

In a good sense of mathematical limit one can say that c is a "projection" of vertical infinite speed or that the very well-known real speed of light **c** is "**essentially infinite**". That assertion makes the 'universality of speed of light' much less mysterious as all other speeds are finite.

The actual speed of light C_{abs}, which turned out to be infinite, we call "Galilean speed of light" while its (finite) "slant projections" to the lines parallel to the line 0B' (the angle θ) we call "Semi Galilean". In particular, when θ = 0, the Semi Galilean speed is the relativistic speed of light c.

In terminology we chose, the Galilean speed of light, which is infinite, is semiGalilean too.
As for the speeds U, which are always finite (but unbounded), we call them the "Galilean speeds". The speed U = u = 0 is also to be called (minimal) Galilean.

More facts on speeds in the complex model one can find in [1].

The above considerations and results incline one to adopt the following hypothesis.

Suppose that the motion in the complex space C^4 obeys (all ?) the ("original") Newtonian rules.

On the other hand its projection (the real part of) to the real subspace (say, $C^4 \rightarrow M^4$) must obey all the rules of the Einstein' special relativity theory (SR) as supported by all the experiments performed in the real part of the complex domain. More precisely, the strictly Newtonian quantities, like distances and speeds, are to be real as the (signed) absolute values of the underlying ["*para-physical*"] complex quantities, while the Einsteinians are real parts of the same (common for both) complex values. That is why the Einsteinian length and speed are always shorter than the corresponding Newtonian whenever the speed u is a nonzero.

To better justify our claim on the primary Newtonian nature of mechanical motion one must reconsider also other physical quantities, such as, for example, time and mass, to discover their, hidden for direct physical observation, Newtonian behavior in the complex domain.

The definitions and facts on speeds, given above in this paper, are necessary for new analysis of time behavior that follows.

Complex Time Transformation; The New Version
The ("old") considerations from section 4 turn out **not** to be consistent with those from section 5.1.

Namely, time transformation, as considered in section 4, is entirely "proportional" to the space transformation as described in section 3.1. Geometrically, this means that the figures (see, Figure 1 and Figure 3) determined by the points 0, B, B' and the points 0, A, B' on (complex) plane of the positions (space points) are similar to figures determined by the time points 0, β, β' and the points 0, α, β' on the 'time plane'. As the consequence of this geometric similarity, to any transformation of a position strictly corresponds the directly proportional transformation of time.

The physical consequence of these geometric facts is **untrue** "invariance of speeds" (u = U) in real and in the radial directions [For the rocket "complex observer", the space metrics in real direction contracts by the coefficient cosθ and, according to Figure 3, "the same" happens (for the same observer) with time in real direction. Meanwhile, both length and time, in the transition from real to radial direction, are invariant in their absolute values.

Thus, the ratio U and ratio u of distance to time in both the directions would be "the same".].

This is in contradiction with formula (7), which, in 5.1 was proven to be true.

Thus, as a consequence of (7) the (true) contracted absolute values (α and |α'| on Figure 3*) of times (but not of corresponding distances) within the rocket in both the directions must be the same so the complex rocket observer moves faster than "*his real counterpart*" (his "shadow").

Here, we mean the 'times' (α and |α'|), as measured by the rocket observer, "who" is considered as "looking" either in relativistic (real) or "Galilean" (complex) "direction". The notion of "time direction" is determined by the trajectory direction, in the complex plane, the rocket is considered to move on.

Nevertheless, the (real) time β (Figure 3*) measured by the **rest** observer, is not contracted at all, while both the times (in both the directions) measured in the rocket are contracted by the same coefficient 'cosθ'. On the other hand, in case of (true) time transformation (only), the complex time, say $\beta e^{i\theta}$, "by the rest observer" has no particular physical meaning.

Let us illustrate this, not so straightforward, phenomena by the following example.

[An illustration of the **true** (final, comparing to the "intermediate" Figure 3) time transformation provides Figure 3*.]

Example: Let us give to Figure 1 the following, different than before, interpretation.

Suppose, now, Earth and a rest observer are located at the origin 0 and the rocket is sent from it at the time moment 0 to a star located, in the real space, at point B. The constant relativistic speed u of the rocket is given by csin θ, while the corresponding Galilean speed U equals ctan θ. With the speed U the (full, complex) rocket moves from Earth 0 to the (full, complex) star, which is situated at complex point B'. The time (as observed by rest observer) that elapsed between start of the rocket at 0 and its arrival to B, equals to the ratio, say, t' = |0B| / u.

Recall, we consider two "different" observers in the rocket. One is real (relativistic) in the rocket situated on the line 0B and the other, "complex observer" is considered to be in the rocket when it goes along the radius 0B'. The first, relativistic observer, "thinks" he is at rest while the long "stick" with the ends 0 and B is passing him with speed (-u). Consequently, he "sees" the ends of the "stick" at points, say, 0A instead of 0B in accordance with the principle of Lorentz contraction.

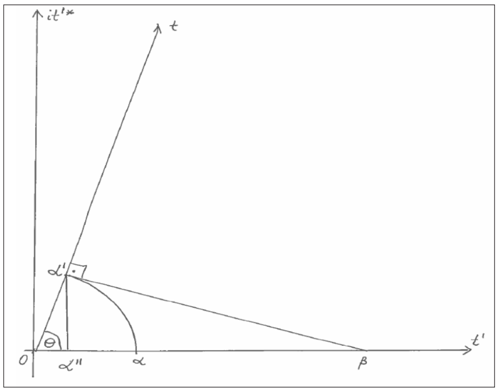
For him, the distance between Earth and the star is reduced by the coefficient 'cos θ' while the speed is that same u. This obviously reduces the time of his journey by the same coefficient cos θ. For this observer's complex counterpart the distance is not contracted as the star is now at B' and |0B'| = |0B| but his speed U is bigger than u by the coefficient sec θ.

The result is then the same for both rocket observers (the real and the complex) i.e., the time is shortened by the same coefficient cos θ in both the directions.

Meanwhile, the time β measured by (one) rest observer at 0 is unchanged and consequently longer than the common time measured by the two rocket observers.

An illustration of the **true** (new, comparing to Figure 3) time transformation provides the following Figure 3*.

Figure 3*



Recall, on Figure 3*, t = tc is the complex variable.

In Figure 3* I presented the complex time plane with the origin at 0. Suppose the (real) time is measured by the rest observer from 0 on and its value, at the given instant, is β. It can be understood as time at which the (real) rocket passes point B on Figure 1 (see the above Example). At the same time epoch, the full complex rocket passes point B' which is the rotation of B by the angle θ. However, unlike point B (and unlike it shows Figure 3) time moment β is **not** rotated by the angle θfrom t'-axis to t_c - axis but instead β is projected to time epoch α' on tc -axis. This orthogonal projection in the time plane illustrates measurement (by rest observer) of time that elapsed in the complex rocket from the beginning of the journey.

Time α for the real rocket observer (the relativistic case) one obtains from the complex time α' by rotation (and **not** the projection) by angle (- θ from complex t_c -axis to the real t'-axis. The composition of the two operations is obviously commutative so first we may transform time β to α (analytically it is multiplication by cos θ) and then rotate by α toward t_c -axis (analytically by multiplying by $e^{i\theta}$). The whole transformation from β to α'(in either order) analytically relies on multiplying β by the product cosθ $e^{i\theta}$, see transformation (13).

Occasionally realize too, that the "Earth time", as measured by complex rocket observer, is (again) orthogonal projection of his complex time α' to the real axis t' and equals α" = Re α' = β $(\cos\theta)^2$

The associated with the above "**twin paradox**" finds a good complex plane's illustration and support by Figure 3*.

This paradox's complex analysis, I plan to develop more in full version of this paper.

Remark. The time transformation, as illustrated on Figure 3*, is consistent with twin paradox's SR analysis. On the other hand, one obtains a contradiction with it once assuming the previous transformation of time according to Figure 3 (and so with the complex Lorentz time transformation (3)). In latter case, the earth time, measured by the complex rocket observer at the complex moment t, equals |t| cos θ [point θ in Figure 3] while "the same" time on earth according to Figure 3* equals to |t| $(\cos\theta)^2$ [point α'' in Figure 3*]. Since, according to twin paradox (so according to SR) the second value is true and not the first, one must adopt the second transformation of time as given by formula (13) and illustrated by Figure 3*.

By the way notice, any measurement of a physical quantity located on a line of a complex plane from a representing this quantity point on that plane is the orthogonal projection from the point into this line.

The Modified Complex Lorentz Transformation

From what was considered in section 5.2 it follows that the time transformation in (3) was not yet quite adequate. Namely, for the "time rotation" (in the complex "time plane"), not only the angle θ is changing (as is changing the corresponding speed u) but also the "time radius"

(t' cosθ) according to changing of the coefficient 'cos θ'. Here, realize that, as on the time plane (Figure 3*) I considered the situation at x = 0 (on the corresponding position plane), t' is calculated from 0 on (i.e., 0 is the vertex of the time "rotation").

So, as |θ| grows from 0 to π/ 2 (|u| → c) the corresponding 'radius of time' decreases from its initial (maximal) value t' (corresponding to θ = 0) to its limit 0 (at θ = π/ 2).

[This means that unlike Lorentz contraction, which only relies on the (regular) rotation in the plane of positions, time dilation (by cosθ) is an essential contraction in both the real and the complex directions. On the other hand the contracted time is 'natural Galilean' while the real time as measured by rest observer may be considered as "artificially extended" because the (corresponding) speeds are "squeezed" in the interval [-c, c], while the distance's signed absolute values are invariant.]

Since the right-hand side of fourth line of (3) must be replaced by the expression **(t' cosθ) exp [i θ]** [in (3*)], it is not anymore 'rotation' (and so, not isometry) in the regular geometric sense.

According to the convention we admit, we replace the complex variables x, t in (3) by new symbols x_c and t_c, respectively in (13), where, as now we assume, x, t [the former x', t'] will represent real axes. Thus, we have

$x_c = x \exp[i\theta]$ and $t_c = (t \cos\theta) \exp[i\theta]$. (12)

In particular, x, t are negative, whenever π/ 2 < θ < 3π/ 2.

Since now the signed absolute value of time tc in the complex θ - direction is t shorten by cosθ, the expression ut in (3) should now be replaced by the (equal in value) expression Ut_c since U, longer by the factor secθ than u, makes up for the contraction of time | t_c | .

According to the common assumption in SR (so in the real domain) we have the relation x = ct as the units of time and length can be mutually converted to each other using c as the factor of change.

However, in both the directions (real and radial), $|x_c|$ is invariant. So, if ±|tc| is equal to t shorten by cosθ, c must increase by secθ and thus we rather have x = ct = C (± $|t_c|$) and for the corresponding complex values:

$x_c = Ct_c$.

For similar reasons one should replace the expression ux / c2 in (3) by Ux_c / C^2 in (13) since

$U / C = u / c$ and $x_c / C = t_c$.

The final modified complex Lorentz transformation takes on the form:

$x_c - Ut_c = x' \exp[i \theta]$

y = y'

z = z'

$tc - Ux_c / C^2 = (t' \cos\theta) \exp[i \theta]$ (13)

Recall, that xc, tc on the left hand sides of (13) are complex, as given by (12) and, obviously, we have

$x_c = |x_c| e^{i\theta}$, $tc = |tc| e^{i\theta}$,

while x' and t'cosθ remain real representing ±| $\mathbf{x_c - Ut_c}$ | and ±| $\mathbf{t_c - Ux_c / C^2}$ |, respectively.

Now (13) reduces to the ordinary real Lorentz transformation (1a) or (2) upon taking the real parts from both sides of the first three rows of (13) and (unlike in the case of previous form (3)) the (signed) absolute values from both sides of the fourth line.

Upon the realizations that sinθ = u / c = U / C, $t_c = x_c / C$ and t' = x' / c we can rewrite (13) into an equivalent nice form:

$x_c (1 - \sin\theta) = x' \exp[i \theta]$

y = y'

z = z'

$t_c (1 - \sin\theta) = t' (\cos\theta) \exp[i \theta]$ (14)

From the above one can confirm legitimacy of our discussion on speeds from section 5.1.

Namely, upon dividing both sides of first line of (14) [so also of (13)] by the fourth line, then taking on the consideration (12) and canceling all the common factors one obtains:

$C = x_c / t_c = x' / (t' \cos\theta) = c \sec\theta$, i.e.,

$C = c \sec\theta$. (15)

Multiplying both sides of (15) by sinθ, one obtains:

$U = u \sec\theta$, (15*)

and thus we have obtained back (7), now as deduced from the modified complex Lorentz transformation (13) or (14).

Conversely, multiplying both sides of forth line of (14) or (13) by identity (15) one obtains the first line of the modified Lorentz transformation. For a fixed value of θ (speed), identity (15) represents a constant, say, C. Thus, one can say that (if θ is fixed) the time transformation is again proportional to the space transformation and the time transformation brings a redundant information. Actually, one preserves (13) as well as (14) if the fourth line will be replaced by the speed transformation (15).

Now, instead of (14) one obtains the following equivalent "space-speed" transformation:

x_c (1 - sinθ) = x' exp[i θ]

y = y'

z = z'

C = c secθ, (16)

or

x_c - Ut_c = x' exp[i θ]

y = y'

z = z'

C = c secθ. (17)

Also, for each θ (so for each u) (15) in the transformations (16) and in (17) can be replaced by the arbitrary speed transformation (15*).

Realize, there is no explicit time variable in (16). However, time can be derived (defined) from (16) simply by arithmetic division. Speed can then be defined without any use of time on purely geometric grounds by means of a position of a physical body in the complex plane of positions. It follows that time is derivable from motion and is not a primary notion. This possibly may shad some light on a (dependent) nature of time. That is why, the space-time complex model C^4 may be reduced to the para-space C^3 model [3], as within the latter model's theory one can derive both speeds and time. The source of both determinations is (probably) the imaginarity so, of course, this reduction of dimension to R^3 could not be possible with the real M^4 model.

Mass Invariance in the Complex Space

Pursuing further the idea of hypothetical Newtonian character of any mechanical motion along the radial direction r = x_c consider the difference in behavior of mass for motions in real and in radial directions.

Thus, as it is commonly known, the Einsteinian mass mE of a physical body moving with a speed u (u = c sinθ) in the real direction x equals m0 secθ , where m0 is the rest mass of this body [here, recall the notation: secθ = 1 / √(1 – u^2/c^2))]. On the other hand, the corresponding mass of the same body, as it moves in the radial ("Newtonian") direction xc, should always, according to the Newtonian dynamics, be equal to the rest mass m_0 regardless of any magnitude of the speed. The question is, how it is possible.

To answer it we need to procced with another construction and the only criterion of existence of the constructed objects or properties (here an "imaginary part of the mass" as defined below) is, the achieved in so doing, an inner logical consistence of the so build theory and preserving within it both SR and the Newtonian theories as parts.

Let us now concentrate on the mass problem. Consider motion of the same body in two directions: the real x and complex (radial) xc. Suppose that the (complex) body moves along the radial direction x_c (Figure 1) with the speed equivalent to the argument θ of its position on the complex plane.

During a real time period, say τ = |α| = |α'|, (common for both the motions of rocket observer, Figure 3*) the motion of the body in xc direction produces the displacement from point O to point B' (Figure 1) , while the body, moving during time τ in the real x direction, produces the corresponding displacement from O to A. Together with the imaginary displacement from A to B' all three displacements form the right triangle OAB' (Figure 1).

As we later will argue, the same momentum (along x and x_c axis) as produced by the same force's magnitude during the same time τ, produces two different speeds u and U and thus two different displacements OA and OB'. As a consequence, the proportion of masses ("along" OA and OB') should be expected as the inverse to the proportion |OA| / |OB'| of the displacements. How does it work ?.

[**Remark:** Unlike speeds and masses the momentum (and also any force) of the body, in both the directions, is the same. As it can be shown, the 'momentum in the imaginary direction' reduces to zero (as composed from two opposite terms which cancel each other). To show it, the independent considerations from this section in below could be helpful.]

It may be expected that the three masses (inertias) satisfy the geometric Pythagorean rule. However, as one can see, more inertia affect less displacement (at "the same time") so the inertia associated with the displacement OA is larger than the inertia associated with OB'. In our "right triangle" (in "complex plane of masses") the hypotenuse is shorter than the side OA along the real direction.

Since, in this case, the "triangle's sides" are the masses (not lengths) we still may expect the Pythagorean rule is satisfied provided square of the mass for motion in the imaginary direction is negative. Thus, the inertia (mass) for the motion in imaginary direction must be imaginary. As it will be shown below, under this assumption, the, say, '**generalized Pythagorean rule**' for the "masses right triangle" with one of the "sides" being of imaginary "length", can be satisfied if the *absolute value* of the imaginary mass equals m0 |tanθ|, which, here, is directly proportional to the length of the side AB'.

[The last assumption preserves the "Pythagorean character" of the "masses triangle" OAB' although in the generalized sense only.

At this point, however, recall that, "normally", in the complex plane the square of length |OB'| equals sum of positive squares of sides OA and AB' so that the length of AB' is real (as the imaginary part of the corresponding complex number - here the position of B' of the considered body). The imaginarity of the "length" |AB'| allows to switch the roles of the sides (related to the masses) OA and OB' according to the rule "*larger mass implies smaller displacement*".

Also, larger mass implies smaller speed (see section 5) as achieved by a hypothetical preceded action of the same, for both directions, force. The latter is the consequence of the times

τ = |α| = |α'| equality in both the directions so the time derivatives from equal momenta are equal.]

The "*(hyperbolic) complex plane of the masses*" that now we encounter is then different than the usual circular complex plane and the "absolute value" in such a plane is slightly differently defined.

This "peculiar situation" happened because we introduced a relatively new concept of "imaginary physical quantity" – the mass, in particular. We analyze this notion closer in next subsection.

The concept of "imaginary physical quantities" is known in literature but rather not in the (classical) framework we put it in this text. Mostly the 'imaginary mass' is considered in association with some properties of quantum fields and associated with them elementary particles [9,10]. One of the first such a field discovered through theoretical considerations that was recently confirmed to exist, is the Higgs field. Some more general cases of such fields (tachyonic fields) were introduced in [9,10]. However, existence of the expected particles (tachyons) as produced by those fields found, up to date, no empirical confirmation (see, related [11])). In nowadays it is customary to believe that some fields may have the imaginary mass but not any particles have. Besides, in literature, there is no classical interpretation of an imaginary mass. The 'imaginary mass' of the fields is defined in such a way that squares of such masses are negative so, at this point, it coincides with the concept here presented.

The need for a distinction between various physical quantities generally defined as "imaginary" emerges. Such a concept occurs in physics frequently in association with complex numbers. Here recall, that the imaginary part of the underlying mathematical quantity, say, A + iB is the real number B and in vast majority of cases the associated physical unit is real as well.

As a typical example consider the complex resistance (impedance) in the electric circuit's theory. So that the impedance Z is defined to be complex quantity and we have $Z = R + j X$ ($j^2 = -1$). Notice, however, that the imaginary unit j here is used to implement the mathematical tools rather than truly define an imaginary physical quantity. So, in this case, no additional physical substance is added. The "imaginary" part of the impedance i.e., the reactance X is still a real quantity and its physical units are the same 'ohms' as the units of the resistance R. Therefore, square X2 of reactance is always a nonnegative real quantity.

Much more such examples from classical as well as quantum physics could be given.

So, return to the case of Higgs fields where square of the mass is negative. In this case it looks that the square $[*B]^2$ of the 'imaginary part' of the underlying "complex physical quantity", say, A + i [*B] is negative while square of its "arithmetic part" B2 is positive since B is a nonzero real number.

Thus, one arrives in a second application of complex numbers to physics where the physical units (here the mass measured in, say, ['i kilograms'] = [*kilograms]) of the 'imaginary part' of the mathematical model are "imaginary" themselves.

One important aspect should now be stressed. The imaginary numbers are mathematical objects while the 'imaginary physical units' (producing real negative squares of the units) may possibly be considered of the **physical** origin and nature.

One of the implications of this conclusion could be non-associativity when "multiplying" imaginary numbers by the imaginary physical units (from now on shortly called "units").

If to consider the notation iB[i kg], where B is a real number and kg is the ordinary real kilogram, the two i's in last expression should not (according to the rule we now admit) be multiplied by each other. Thus, according to that rule iB[i kg] ≠ - B kg.

The imaginary part B[i kg] of the above considered expression is not, however, real anymore but its *imaginarity* is rather of physical (not of mathematical) roots.

Its square is anyway real negative i.e., $(B[i\ kg])^2 = -B^2 kg^2$. In this way one could possibly understand the imaginary mass B[i kg] of the tachyonic (in particular Higgs) fields or tachyons, if they would exist.

Returning to the expression iB[i kg] one, unfortunately, must admit that this is not anymore (a part of) a pure complex 'number' only. What kind of the object is it is not clear for me yet as I could not find any analysis like that in literature.

One would say, this object is a (expressed in a 'language' beyond a mathematical language) fusion of the mathematical object 'i B' with a physical entity '[i kg]'. As the so described, we will try to apply this 'combined entity' to model the 'imaginary mass' that we mentioned in section 6.1.

The latter concept as, to an extend, a concept of "classical" physics, treating imaginary masses of physical macro bodies (not necessary the fields) seems not to have any predecessors in existing literature, at least not in the considered in this paper framework and not according to my best knowledge.

Remark: Shortly, the concept of the 'imaginary physical quantity' may be characterized by saying that: unlike with regular imaginary quantities, as met in complex analysis applications, the "absolute value" [so the 'true measure'] of the 'imaginary physical (as opposite to mathematical) quantity', is "imaginary" (in, say, a (para)physical sense).

As far as now, I only considered the imaginary masses. It seems, however, other physical quantities may also have their imaginary counterparts. For example, it is well known procedure to substitute in the Minkowski M^4 model the 'imaginary time' τ = it in place of the real time t in order to regain the Euclidean R^4 space and so to proceed with the "Euclidean special relativity". It is not clear for me to the end the physical interpretation of the "time" τ (also, independently, obtained by the 'Wick rotation'). Do we measure it in '[i seconds]' i. e., in imaginary units and what is the physical meaning of it ? Or is it only the result of mathematical formal substitution τ = it , that simplifies some calculations but without saying a lot about the underlying physical meaning or nature ?

Other, potential, way to encounter an "imaginary distance" [as measured in "[i meters]" that might, eventually, be introduced in a similar way (with a similar meaning) as the imaginary mass, here considered], could possibly be provided in association with the 'Schwarzschild metrics' [1] for the space in gravitational field. Namely, in such a field, the original distance of a body to the star is increased, so after extending the real space to the complex, one could expect the radial distance (in the interior of the [hyperbolic] complex space) to the star was invariant. We might achieve it in the similar way as in the case of the 'Newtonian mass invariance' by properly introduced 'imaginary distance' which would counteract the real space extension. Eventually, this might help to understand the phenomenon of the gravitation. This, however, goes beyond the scope of this paper. Anyway it suggests existence of other (possibly many) 'physical imaginary units' that are not of mathematical origins.

In order to stress the nonmathematical origin of the imaginary physical units and to avoid the underlying suggestion of the origin just obtained by multiplication the real unit by the mathematical unit 'i' we change a bit the notation. Suppose then that μ is a real physical unit such as kilogram, second, meter etc … .

Instead of writing [i μ] (which is an unsplitable quantity rather than the arithmetic product) we rather use the notation "[*μ]" that will be spelled "star μ" .

Assume we are using a 'language of physics' that properly contains a language (a part of it) of mathematics. Now we may claim that such symbols like " * ", "μ" and "[*μ]" do not belong to mathematical language (i.e., these symbols have no mathematical meanings) but still belong to a language describing physical reality. In such a way we treat such objects like [*kilogram] as "purely physical" which, nevertheless, may be combined (in the overall "extended language") with mathematical objects and operations such as multiplication and taking square. Besides, for the new notation, the defining property of these units is the following relation:

$[*\mu]^2 = - \mu^2$, in particular $[*\text{kilogram}]^2 = - \text{kilogram}^2$. (18)

Conceptual benefit of the new notation, apparently, is achieved. Namely, when talking about the *absolute value* [i.e., the 'actual measure'] of such quantity like '5i [*kilogram]' we avoid the inconvenient notion of "*imaginary absolute value*" (i.e., when expressed in | [ikg] |) in a sense of the presence in the expression an arithmetical imaginary quantity that involves i. Instead, we have | 5i [*kilogram] | = 5 [*kilogram]. So that we obtain, as the actual measure, *physical* units (5 [*kilogram] and mathematically just the real number 5) which are related to the kilograms by the second formula in (18).

The measure (an *absolute value*) is then a real number multiplied by some, newly defined, (possibly unobservable) physical quantity. Actually, no matter what quantity is it but the number of the units is real.

A. Return to the problem of the masses redistribution in (regular) complex plane.

The overall (Newtonian) complex mass mN as the measure of inertia for the motion along the radial direction xc in the complex plane (Figure 1) can now be redistributed according to the formula:

$mN = mE + im^*$, (19)

$[(m^*)^2 = - m^2$ for any mass m]

where the mass $mE = m_0 \sec\theta$ [kg] is the common relativistic (real, Einsteinian) mass for the motion along the real axis x as measured in kilograms.

$m^* = m0 \tan\theta$ [*kg] is the corresponding imaginary mass measured in the 'star-kilograms' [*kg] according to the notation introduced in previous section. Recall that $[*kg]^2 = -kg^2$.

The 'absolute value' of the complex physical quantity mN one obtains as

$|m_N| = (m_E^2 + m^{*2})^{1/2} = [m_0^2(\sec^2 \theta - \tan^2 \theta)]^{1/2} = m_0$, (20)

shortly, we have:

$|m_N| = m_0$.

Concluding in words, the magnitude $|m_N|$ for the Newtonian mass for the motion in the (complex) x_C- direction is independent from any speed, as represented in (20) by the angle θ, and is the same as the rest mass m0 of the given body, even if the Newtonian mass m_N itself is complex.

The last statement fully supports the hypothesis on the Newtonian character of motion in the complex radial direction x_c, while the relativistic character of motion in real direction is preserved too.

The formula (19) with all the above assumed meanings should, probably, be added to the complex Lorentz transformation (13) as the additional row.

B. The Newtonian rest mass invariance, as expressed by (20), put some new light on the Newtonian (Galilean) nature of speeds and their unbounded character, when considered in the underlying complex direction x_c.

The following reasoning is based on the **assertion** that the body's momenta absolute values p_x and p_{xc} in the real and the complex directions, respectively, satisfy $p_x = p_{xc}$.

The assertion follows the fact that square of the imaginary momentum is zero [Realize that, in this imaginary direction, square of the momentum increment by speed and the square of its increment by the mass are equal real quantities (terms) with opposite signs.]

Recall, that the relativistic momentum i.e., the momentum in real direction, equals

$p_X = m_E u$

so it is equal to ' ($m_0 \sec\theta$) u ' where u is the relativistic speed in direction x.

Since, in the complex (Newtonian) direction x_c, the absolute value of the mass is m0 and, by the assumption that $p_x = p_{xc}$, we obtain:

$p_x = p_{xc} = m_E u = (m_0 \sec\theta) u = m_0 (\sec\theta u) = m_0U$,

where $U = u \sec\theta$

is the so defined (unbounded) Galilean speed [recall again the more typical notation

$\sec\theta = 1 / (1 – u^2/c^2)^{1/2}$)].

This speed was also defined in section 5 as $U = u \sec\theta$ on purely kinematic bases.

Here, we just applied in (21) the associativity of arithmetic multiplication to obtain the corresponding (para)physical rule. Other nice form of formula (21) can be expressed as:

m_E u = $|m_N|$U = m_0U. (21*)

[Anticipating, by analogy, we may presume that, moving along the real x-axis, **electric charge** has an increased (by the coefficient secθ) [absolute] value along the unobserved direction xc, corresponding to Galilean speed, while its value in the real direction is invariant ("minimal"). An underlying, hypothetical, additional "imaginary electric charge" 'iQ' (probably) might somehow be identified with a "magnetic charge" which produces a corresponding magnetic field.

Consequently the magnetic field B could be considered as an "imaginary electric field", say, B = i E' (or [*E'] ? ?)

So an imaginary part of the charge Q' would produce an imaginary electric field iE' (i.e., the magnetic in real space) ??.

Hypothetically, there might be a kind of "**skew symmetry**" between laws of mechanics and electrodynamics.

As a model (of each coordinate straight line of [originally] R^3) that would reflect such symmetry (within a common theory) could be used the space of square 2x2 matrices with the rows considered as representing complex quantities. Possibly, no use of quaternions in this context.]

Remark (A Hypothesis): Suppose, a **photon**, that moves in the real positive direction x with the speed c has a mass μ. It is clear that photon's, and any other electromagnetic waves', mass transformation is different than that for the "regular" physical bodies moving in the real direction with speeds u less than c whose mass transformation is given by (19). Since light moves in its 'natural direction' (that is parallelly to the imaginary axis (say, with θ = π / 2)) with infinite speed, one might expect the photon's mass associated with that direction is zero. Also, by continuity argument, one may expect, its absolute mass in any intermediate direction θ is less than μ but positive.

As in the case of other physical bodies, for such a mass decrement an occurrence of the imaginary mass is responsible. In order to achieve a good inner consistence of the constructed theory, assume that, unlike in the case of a regular physical body, this imaginary mass's magnitude (measured in [*kg], see above) equals to

$$\mu^* = \mu \sin\theta \text{ [*kg]}, \quad (22)$$

where, since $[\text{*kg}]^2 = -\text{kg}^2$, one obtains:

$$\mu^{*2} = -\mu^2 \sin^2\theta \text{ [kg}^2]. \quad (23)$$

In this, 'electromagnetic case' one obtains, instead of formula (19) the following formula for the considered photon's complex mass in a θ direction:

$$\mu_N = \mu + i\,\mu^*. \quad (24)$$

In accordance with (22) and (23) the absolute value of the above mass in a θ direction is given by:

$$|\mu_N| = \sqrt{[\mu^2 + \mu^{*2}]} = \sqrt{[\mu^2 + (-\mu^2 \sin^2\theta \text{ [kg}^2])]} = \mu \cos\theta. \quad (25)$$

It is then consistent with the fact that the absolute value of the photon's mass in its natural direction θ = π / 2 reduces to zero.

Realize, that if one would define the photon's "momentum" in a classical way as 'μ c' then such momentum '| μN | C' would be speed invariant, being always the same even for the infinite speed (θ = π / 2).

In the last case the momentum could be well defined as the limit for θ → π / 2.

However, the "energy" seems to be higher (by the factor secθ) than, say, μ c^2 ?.

A Word on Recovery of the Original Newtonian Equations in the Natural (Complex) Directions

At first stage I will consider the Newtonian equations as differential equations of first order only, for simplicity setting on the left hand side (in both relativistic and Newtonian case) only first derivatives of momenta over time. Here, realize that according to the assertion from section 6.3 B the momenta in x and in x_c directions are equal in their absolute vales ($p_x = \pm |p_{xc}|$, where the sign adjustment is obvious). Also, according to the reasoning from section 5.2, the "signed absolute values" of times, say, α, ± |α'| on Figure 3* in the two directions also are equal (as before, we denote their common real value by τ). These two equalities imply the equality of the derivatives:

$$dp_X / d\tau = d(\pm |p_{Xc}|) / d\tau \quad (26)$$

This, in turn, yields equality of the forces acting either in real direction x or in the complex direction x_c.

However, the two sides in (26) are differently expressed.

Namely, for the relativistic case, we have

$$dp_X / d\tau = d / d\tau \{m\, u\} = d\,m / d\tau\, u + m\, du / d\tau \quad (27)$$

since the relativistic mass m is the function of time τ.

Unlike that, for the right hand side of (26) we recover the "ordinary" Newton equations as we have:

$$d(\pm |p_{Xc}|) / d\tau = m_0\, dU / d\tau \quad (28)$$

since m_0 does not depend on τ.

In such a way in (28) we obtained back the left hand side of the classic Newtonian equations, this time for the arbitrary high (but finite) speeds U.

Having measured the initial conditions for the real x directions we may immediately transform them into the corresponding "initial conditions" for the complex xc direction. It is enough to multiply both an initial position, say, x_0 and an initial speed u_0 by secθ. The angle θ is always given by: θ = arc sin (u0 / c), where c is the ordinary (real) speed of light.

Having obtained, in classical way, some solution for an initial value problem associated with the "equation" (28) or its full version as the second order differential equation [now the speeds at a moment τ can be arbitrarily high] we multiply both positions and speeds (being functions of τ) by cosθ(τ). The arguments θ(τ) are always obtainable from the formula:

$\theta(\tau)$ = arc tan $(U(\tau) / c)$,where $U(\tau)$ is the 'speed part' of a Newtonian particular solution.

In such a way we will obtain back the real, empirically verifiable, solutions of the original, relativistic problem.

The Newtonian theory is, in such a way, extended to arbitrarily high finite speeds and closely (not only approximately) associated with the SR version of the high speeds calculus.

Obviously, the described above reduction of relativistic motion to the Newtonian motion may be expected to give a **significant simplification of** underlying **calculations**.

Moreover, realize that having the complex (values) model in dealing with the Newtonian theory we actually only consider (signed) absolute values of the underlying complex quantities. Until we get a final solution we may just ignore the arguments $\theta(\tau)$ for any time τ.

Thus, the classical problem in the C^3 complex model naturally reduces to the real R^3 model when the positions or velocities are considered. To obtain the proper R^3 space model for the extended Newtonian theory we obviously must admit negative real coordinates (the signs adjustment).

Therefore, for the transition from the first coordinate $x_c = x + ix^*$ of the complex, say, position $(x_c, y_c, z_c) \in C^3$ into the first coordinate x1 of the corresponding real position $(x_1, x_2, x_3) \in R^3$, we assign to each complex first coordinate x_c of a position the corresponding real position's first coordinate $x_1 = \pm |x_c|$, where the sign of x_1 is given to be the same as is the sign of x.

Moreover, $x_1 = 0$ if and only if $x = 0$.

The transitions $y_c \rightarrow x_2$ and $z_c \rightarrow x_3$ are defined quite analogously.

Notice too, that no "regular" physical body (a classical point-particle) is situated at a nonzero point on any imaginary axis unless it is a photon or the like (that involves infinite complex speed).

For the C^3 space of complex speeds Uc, however, we should apply (in each of the three complex coordinates of the C^3) the formula $U = \pm|U_c| = u \sec\theta$, where, unlike in the case of positions,$-\pi / 2 < \theta < \pi / 2$. Thus, the Newtonian (Galilean) speed U has the same sign as the relativistic u

Obviously, if $\theta = \pi / 2$ then $U = C_{abs}+ = +\infty$ and if $\theta = -\pi / 2$ then $U = C_{abs}- = -\infty$ i.e., we encounter the absolute Galilean (infinite) speed of light. This procedure is essentially the same for each of the three velocity's coordinates of the C^3, where all the three angles (the arguments) $\theta_1, \theta_2, \theta_3$, corresponding to the speed's coordinates u_1, u_2, u_3 , are applied (with $-\pi / 2 < \theta_i < \pi / 2$, i = 1, 2, 3).

Now, in both, relativistic and classical, cases we may consider two separate R^3 models. In each of the two cases, one of the R^3 models applies for positions and the other for speeds (so as the common model one may consider sort of 'phase space': R^6 as R^3 x R^3 , here with speeds instead of the usual momenta). In relativistic case the coordinates are the real parts of the underlying complex coordinates of the speeds in C^3 while in the Newtonian case they are the (signed) absolute values of the same complex.

That may be considered as the fundamental relation between the relativistic and the Newtonian versions of the same (complex) mechanical motion.

In light of the above it is natural to predict that the **list of forces** usually present on the right-hand side of any Newtonian equation is the same, no matter if the left-hand side is given by (27) or (28).

This is the fact that any force in the complex x_c direction has the same magnitude as the corresponding force in the real x direction. This follows from (26). Based on the above now one can set up the Newtonian equations for all the forces that act in the complex x_c direction and so transform any relativistic problem to the corresponding classical.

As mentioned, the obtained classical solutions $X(\tau) = \pm |x_c(\tau)|$ and $U(\tau) = d / d\tau\, X(\tau) = \pm|U_c(\tau)|$ can be transformed back to the corresponding, empirically verifiable, relativistic solutions $x(\tau)$ and $u(\tau)$ by projection of the ['complex motion'] into the real subspace (i.e., by multiplying both of the classical by $\cos\theta(\tau)$, where $\theta(\tau) = \arctan(\pm|U_c(\tau)| / c)$).

Further, detailed, development of this set of problems is out of scope of this work.

My intension only was to sketch the essence of introducing Newtonian equations to the C^3 model in order to show that both the theories (classical and the SR) are parts of a wider theory whose model is the Euclidean C^3 (and, possibly not C^4, see [3]). For both the theories taken separately the models were reduced to the real R^3.

As it may follow from [3] (see, Appendix 3) a third theory that must rely on the full complex description (not reducible to R^3) is possibly the **quantum mechanics** (see a free particle' model as the small "vibrating stick" [mathematically it is the harmonic oscillator] in the complex plane [1]).

The three mechanics might possibly be unified in a wider theory of the C^3 (para)physical model?.

However, I do not see, at the moment, a proper relation of even classical electrodynamics with the ideas above presented.

Maybe a model that, additionally, would also be the model of electrodynamics theory might be the C^3 x C^3 space (configurated as $C^2xC^2xC^2$ with each C^2 representing a real straight line) with a "skew symmetry" between mechanical and electrodynamical phenomena?.

Remark: As mentioned in the new version (13) of complex Lorentz transformation may be replaced by equivalent version where, with the invariant space transformation, time transformation is replaced by the following "speed transformation" (for the first coordinate):

$U_c = (u \sec\theta)\, e^{i\theta}$,

see formula (11). The duality of the latter with (13) is evident. The semiGalilean speed of light transforms analogically i. e.,

$C_c = (c \sec\theta)\, e^{i\theta}$,

see (10). So the time transformation follows this speed transformation.

However, in this case, we actually arrive with C^6 as C^3 x C^3 model.

Appendix

A. When dealing with the above considered "paraphysical" objects, (which, mathematically, are described as immersed in C3 - R3 part of complex "para-space" C3) as well as with their properties, the question that may arise is: "**do they *really* <u>exist</u>**"?.

Both, the question and an attempt to answer it should not be considered as belonging totally to the existing physics as they in a sense go beyond the class of objects the contemporary physics is dealing with. The problem, to a large extend, belongs to ontology and metaphysics and as such its possible solutions are of hypothetical nature depending on particular ontological and metaphysical theories and approaches [12-16].

Basically, in most of the contemporary ontology, all the objects of any interest are subdivided into "concrete" and "abstract", where the "concrete" are those that are, say, 'physical'. Most of the philosophers (although not all) agree that the concrete objects exist while disagreement is about the nature and existence of "abstract objects" such as mathematical objects, mental (feelings, impressions or thoughts), art or literature's fictious characters etc …. Basically, those who deny the latter's existence are labeled as 'nominalists' and the other 'Platonists'. For the latter's view, see, for example, [16-20].

Some, more radical, Platonists deny existence of objects that "do not exist" claiming that if '*something*' is a subject of our discussion it must exist that way or other (not necessarily the physical way of being). So, according to them, talking about "nothing" is rather impossible [21].

The ontology is not yet a finished nor unified enough project and one only can refer to one or other approaches to its problems and various obtained solutions often contradict each other [13]. Therefore, the question that in this situation naturally arise is if we really must answer questions on the existence [22].

The literature on ontological problems of existence is extremally wide and it's hard to find any univocal answer to a question whether a given class of objects "does exist" [22,21].

As for my own view on the subject (especially on possible existence of the "paraphysical" objects as located in C^3 outside R^3) let me propose what follows. Of course, I am not able to give an answer that would satisfy everybody and, finally, I must refer to one's intuition or just believe. The question is on 'existence'.

Here, recall different meanings of the phrases "exist physically" (in the so called "real world") and "exist as an abstract object" such as number 3 or our feelings or a (platonic) idea separately or in mind.

Have we, in this case, an '*existence*' in four (possibly not the only) different meanings or it is one 'existence' but in four different domains ("places").

This is a typical tendency among people, especially non-philosophers, to reduce different types (or domains) of existence to one, especially to reduce all to the physical existence. The reason probably lies in serious difficulties in one's understanding other than "physical" possibilities. The so called "real world" (read: 'physical' world) is perhaps the best understood.

On the other hand, its existence is uncertain for those who follow various philosophies of the subjective idealism. According to Berkeley we only know our impressions or only impressions exist for sure. This philosophy has many followers [24-26]. The empirio-criticism mostly created by Avenarius and then developed by Mach to a large extend shares that point of view impacting next generations of philosophers such as those from Vienna circle and, more generally, the logical positivism and logical empiricism of twenty century [27-29]. The point of view, that impressions and consciousness are primary and physicality (the matter) needs special proof for the existence or does not exist at all (being of mental nature only) is still present up to nowadays (see, for example, [30]).

The truth, probably, is in between and (according to the pluralistic solution) various objects exist in various domains such as the physical world, mental world, mathematical (as a part of the platonic), the world of possible art, literature, values, also, finances, law and others.

The next problem of general nature, besides the existence, is a proper understanding the notion of "reality" which means "all that (*really*) exists". What, actually, does it mean ? Here is a basic question: how do we know that a considered object appearing as our thought or name in a given language "does" exist (for example, outside of our mind).

At this point I would rather avoid to narrowing the answer to assertion: 'It exists because it is a part of physical world' or in its negative version avoid to say: 'It does not belong to "real world" and therefore *not really* exists'. The general sense of notion of 'existence' should not be dominated by its specific (here, physical) sense.

At this point some would prefer more general criterion for the existence in its 'true generality'.

It is reasonable to relate **existence** of an object, say X with any kind of its **interaction** (not necessarily dynamic) with (some properties of) other objects, say, Y_1, Y_2, … that 'previously' were proven to exist. {Consider this statement not as a kind of **definition** but rather criterion of existence as later will be applied to our objects in the $C^3 – R^3$}

In my opinion, such a theory of existence [as containing the above criterion of the existence] must contain some primitive notions and assumptions, (that, in general, varies in a process of theory's creation) even if not everybody would be very happy about this. In such a context, one could talk about existence with respect to other existence [17]. In particular, this relation may be mutual. The possible "interactions" can be dynamic, when physical objects are considered, or static (relations) like among mathematical entities or mixed among some mental. A common in mathematics saying like: "act by this matrix on that vector field" is an example of static relation which on the mental level looks like (or is) the dynamic. For better generality it is convenient to talk about "relation" rather than "interaction".

According to the above, something 'exists' if one cannot ignore it when considering its existing "neighborhood".

Other example: one should not ignore complex numbers when considering the class of all polynomials with real coefficients and the associated algebraic equations.

In politics, rulers cannot just ignore some "dangerous" ideas even if they appear not to be 'physical'. They nevertheless exist in the above sense and often require an action. In physics, if some phenomena are "describable" by a differential equation, existence or nonexistence of some solutions (with given mathematical properties) of the equation is, actually, of the physical matter even if the solutions physically "do not exist" nor "exists" the equation.

Generally, I would adopt the convention that 'existence' of an *object* means its indispensability for explaining any property or behavior of whatever that was previously considered to exist.

Let me here give an example from the most fundamental (but classical mathematical logic) theory in mathematics i.e., from the Zermelo – Fraenkel system of set theory [31].

To build the subject of this theory one had to determine its existence i.e., existence of sets. For this aim one of the theory's axioms asserts the existence of at least one set (empty set for example). From other axioms one can derive the existence of some additional sets based on known existence of some other sets. Having an increasing supply of existing sets one can prove existence of other sets that are related with the existent ones by showing that the "new (say, hypothetical) objects" satisfy the 'axiom of substitution' and therefore are existing sets too. The substitution axiom only says that the new 'hypothetical sets' are related, in a specified way, to the existing sets by satisfying a relation [function, in this case] between the sets elements, where the relation is described by an underlying formula [31]. This relation, if satisfied, gives to the hypothetical objects the ontological status of being a set. This methodology of the existence proving apparently agrees with the one we proposed above. Notice, that according to the modern mathematics' methodology, all other mathematical beings can be reduced to the set theoretical beings. Those beings according to pretty many (but still minority) philosophers form the "third realm" independent, as for their ontological roots, from the physical and psychological realms.

But then realize that among physical beings the methodology of the existence proving is even if not identical then still similar to that in mathematics. To prove something physical exists we need to show that it is related to something that already is known to exist, like for example our physical instruments or even human senses. In the latter case our conclusion on existence of whatever will be valid if the senses of several other persons will confirm our own observations to eliminate the subjectivity bias.

Returning to our problem of the existence of the paraphysical reality that we deal in this work the (future) methodology for eventual showing existence [if that is true] of objects lying in, say C^3 - R^3 would, mostly, rely on showing any relationship [especially the causal] between those hypothetical beings and the existing physical. The rational consistency of the system described as the C^3's theory also has its important role, but the consistence of the system indicates at least logical (formal) relations between the hypothetical para-physical realms and the physical which, at least, stands as a set of arguments [not yet the proofs] for an existence of the para-physicals.

Here realize that the Newtonian theory (of para-physicals) in the complex space naturally explains paradoxes (such as the Lorentz contraction or universality of speed of light) of SR in the real subspace R^3 or in the real M^4 . This (in the light of the previous considerations) already indicates existence of the "Newtonian" objects in $C^3 – R^3$. The question that remains is, say, "degree of their physicality" as well as physicality of the whole space described by C3 or C4 .

Generally, the problem of "physicality" and its possible distinction from 'mental existence' is subtle and certainly nontrivial. It arises at full extend in quantum physics.

Recall its historical roots; seventeenth century. With the (see, the *Meditation* by Rene Descartes,) famous "psychophysical problem" various philosophers through that and next centuries tried to prove existence of physical world based on mental, while other, like Marxists and other materialists tried the opposite from physical derive the mental [32]. Since the beginning to nowadays none of the proofs, as "given" by many, were satisfactory. The problem may lie in the fact that simply no such proofs (of either way of the dependence) exist.

As some contemporary philosophers consider the two (actually very obvious) statements on the existences are logically independent; the "phenomena" very well known in mathematics (see, for example, the 'continuum hypothesis' in set theory or the 'fifth Euclidean axiom' in elementary geometry) [33].

But, instead of trying (as a remedy to this logical difficulty) to reduce physicality to mentality or in the opposite way, it is possibly better simply to **assume** both physical and mental existences as the two distinct interacting but "logically independent" (possibly separately created ?) realities. The 'monistic solutions' are those I, personally, would prefer to avoid.

But, as for my own view, instead of dualistic alternative, I rather would opt for pluralistic solutions as they seem to me more reasonable. So, in addition to the two beings (physical and mental) one can, as we did above, assume separate existence of mathematical objects and properties on platonic or other bases as well as, on similar bases, existence of world of values and possibly others.

NOTICE: Nevertheless, I presume that some solution that I would call "***para monistic***" may still be formulated. But first realize that being in a relationship with something that exists is a proposed criterion for the existence but still not the definition. In turn, the existence's definition as formulated below does not explain phenomenon of existence so the definition and the considered above criterion of satisfying that definition should be taken together.

Let now consider the simple **definition of the existence** based on the set theoretical relation of belonging '$\in$' to an existing set or a class.

According then to the proposed para monistic viewpoint consider one "space" **(say, the "domain of discourse")** which may be called "EXISTENCE" or "BEING" which contains, as a class of sets or classes, all possible domains such as 'domain of physical objects and phenomena', 'domain of mental beings', 'domain of formal beings' (including the "mathematical") and a finite (or countable ?) number of other domains already mentioned. (At this point notice, that what usually is considered as 'domain of discourse' is the union of all the domains here considered.)

The "BEING – space" may be considered like one mathematical concept of set (or possibly other similar concepts like, for example, the mereological domain) and the domains are related to it according to the belonging '$\in$' relation. Assume that the BEING is not an empty set.

The considered nonempty SET [the BEING] is one, so the name of such developing ontological solution could be "para monism".

Consider the potential [depending on a philosophical viewpoint] elements of the BEING as the classes, say Ph, Ps, M and F plus some other classes, say V, A1, A2, … . The classes are classes of physical objects (Ph), of psychological or mental (Ps) , of the Platonic forms (F) of the mathematical M [with, possibly, $M \subset F$] of values V and other, say A1, A2, … .

This list of the classes is simplified, of course, as such classes like class of events, propositions, fictious objects and possibly many other, for simplicity, were omitted.

Such a set theoretical model is supposed to work for different philosophical systems.

For example, to say: "mathematical objects do not exist" would be the same as to say:
'$M \notin$ BEING', where M is the class of math. objects. In fuzzy sets settings one also might say that the membership function of M is less than 1 (a "partial existence" ?) or 0 (nonexistence)]
The general definition of existence of some 's' would be the following:

The Definition: Denote the BEING by E and assume E is not the empty set.
Something that has a name 's' does exist iff there is an A such that $A \in E$ and $s \in A$.

If 's' does not satisfy above definition then 's' is an empty name (no semantics) and

" there is no s " [itself]

even if the name 's' exists [like a 'round square' does] as an element of some language.

This may be considered as a proposed definition of the 'negative existence'.

To decide whether $s \in A$ for some $A \in E$ the above criterion of being in relationship with something already existing should be satisfied.

Realize too that the nihilistic philosophy can be described by the statement: E = 0, where here 0 denotes the empty set.

Now, when talking about "kinds of existence" it should always be indicated to which "domain" a given object belongs which would correspond to the statement "what is the object's **nature**"?.

As mentioned, the question that remains in this set of remarks is the problem of possible physicality or "*partial physicality*" (?) [para-physicality] of the objects that we introduced in this paper. More on the general problem of the existence is out of scope of this Appendix.

B. According to the conclusion (a criterion for the existence given above) from 'Part A' the "objects" [lying outside the "regular" physical space (as described by R^3) but still lying in "something" mathematically described by the set of points ("positions") in $C^3 – R^3$] do **("somehow") exist** if they 'interact' (at least "mathematically") with the existing physical objects from R^3. The interaction, so the existence, is possible since the objects of our present interest are endowed with physical units such as meter, second, dyne or erg but in the complex numerical setting. Some units, however, are "physically (so essentially) imaginary" like, for example, [*kilogram] and, consequently, the units of "imaginary energy" [*erg] (see section 6). Thus, the units and the corresponding objects are somewhat different than the regular real.

Now the question is about their (full or partial) physicality. So, what is the nature of these objects and of the (*physical*) space C^3 ? Are they physical? or "merely" mental or mathematical or it is some other kind of objects ("paraphysical"?) that needs special discussion and eventual definition.

We will try to address this question in what's comes up. The objects introduced in this paper differ from the objects treated by physics or known (in the macroscale) from everyday experience by two features.

Firstly, even when they are of macroscopic size they are not given by human senses.
Secondly, they cannot be directly recognized by any physical instrument.

(On the other hand, they can be well understood by means of the very well-known mathematical model and the associated consistent theory that preserves (possibly all) the known physical theories of regular physical objects and phenomena.).

So, as mentioned, first problem of their, possibly not full, physicality is the lack of sensuality. Here, the answer is rather simple since many physical objects such as electromagnetic waves or objects in the microscale are not given by human senses and still are considered to be 'physical'.

As for the second question, direct measurements by physical instruments do not recognize imaginary quantities even those that are measured in the real units such as 'meters' or 'seconds'.

At this point, within the theory constructed in this paper, one sometimes could talk about an
"**illusion of physical instruments**" in a similar way as about the, known from the ancient times, phenomena of 'senses illusion'.

As an example consider the Lorentz contraction. The moving fast body is measured in R3 to be shorter than its original length as measured at rest. From the theory, that may better reflect our understanding, it follows that the length, actually, was not changed and the real reason for the "would be contraction" is the rotation of the body in the wider complex space.

Present, at this point, epistemological problem is the one that divides most of philosophers since the ancient times until now. It relies on the question, what should we trust more (or even exclusively): to experimental observations or to well understanding [the empiricism versus rationalism dispute]. Few philosophers (in particular, Immanuel Kant,) opted for the synthesis of these two big currents of the human thought [34].

Needless to say, that (unlike R. Descartes, or F. Bacon) the two successful creators of modern physics Galileo and Newton, in their philosophical efforts to build foundations for the new science, strongly opted for the synthesis of experimental and mathematical [rational, but not just logical (inductive)] methods [32-35].

In my opinion a synthesis like that could form a well ground for the theory presented in this paper. Based on that, we can, at this point, introduce notion of "indirect measurement of the complex quantities".

The main mathematical basis for that concept is formed by numerous theorems in complex analysis (Cauchy , Bergman – Weil , Bochner – Martinelli and other) which claim and proved that, under weak assumptions, for analytic functions in C^n the set of their values on a boundary (that close some open region in C^n) uniquely determines all the values in the whole region. Such a situation also takes place when the considered boundary is Rn or some of its closed subsets and the open region closed by this boundary is a part of (or the whole) C^n - R^n with, in particular, n = 1, 3, 4. (See also the Paley-Wiener Rn → Cn extension of some Fourier transforms [35-39].)

Thus, if values of some physical quantities, generated by physical instruments measurement, fulfill a proper subset of R^1 or R^3, the "remaining" values in the interior of complex space can be calculated by a proper mathematical formula well known in complex analysis.

In this sense one can talk about '**physical indirect measurement**' of 'complex physical quantities' by measuring the corresponding real physical quantities. [By the way, many real physical quantities' measurements also are 'indirect' in above sense (see, typical measurement of electric current 'i' or various temperature measurements).].

On the other hand, many of "complex physical quantities" can be measured (indirectly) by more elementary methods that follow theory provided in above text. For example, if length of a body at rest is known, the measurement of this length when the body moves fast uniquely determines speed, say, u. Having the so measured u we automatically find the angle θ of the body's rotation (see, sections 2 and 3) in a complex plain as θ = arc sin (u / c). Finally, the body's (unobservable directly) Galilean speed U one obtains as U = u secθ. In the sense here described, the (possibly all) complex quantities are "measurable" by the 'indirect measurements'. As already mentioned, this idea is not new in physics and not only in quantum theories.

To almost the same degree we never directly observe the physical phenomena that take place inside the sole or other stars. Similarly like in the above presented theory all our measurements are reduced to the outer surface of the stars.

Also, any knowledge of the far past (including biology, geology and similar) is based on what we know about the presence i.e., the facts "on the surface of time" (in its long version). And many such indirectly obtained facts has the legitimate citizenship in the human knowledge. Other examples are the psychoanalysis as well as criminology or archeology and so on.

Thus, I don't see the reasons why the above theory of the paraphysical objects in C^3 or C^4 should not have similar or even stronger ontological status. Now, however, the question is not just about the existence but about the 'physicality' and at this point the answer cannot be
a univocal one.

Continue the ontological interpretation of the C^4 - or rather "C^3 – theory". Besides the observability, two related issues may, in this case, cause a kind of doubt for some readers.

First issue is that the considered "paraphysical world" is placed in the 'para-space' having higher dimension (topologically, at least six dimensions) than the regular physical space that is, approximately, R^3.

Here, the situation is, to a measure, similar as in the **superstring theory** which advanced the idea that the vibrations of minuscule energy strings were responsible for all that we observe in nature; these theories only worked, however, in a universe comprising ten or more dimensions, with the phenomena beyond the limits of ready observations [40, 41].

The higher dimensions are very well understood in mathematics. Since about middle of nineteen century geometry of both R^n and C^n (for an arbitrary $n \geq 4$) analyzes in details such the geometric hypersurfaces like k- dimensional (k arbitrary with k < n) ellipsoids, paraboloids, hyperboloids and many other geometric objects [42], which (like the objects I constructed) cannot be visualized but their applications to mathematical analysis and therefore also to outside of the mathematics are tremendously huge.

That geometric theories (and their extensions toward infinite dimensions) do not ask any more about "real" existence of the more than three dimensional surfaces in R^n nor in C^n (for n > 4) even if nobody ever "saw" them. Thus, the contemporary geometry at least provides rationality of what is going on in, say, R^6 or C^3.

The hypothetical beings and phenomena that we considered in this work are then at least *rational* and as it also follows consistent (!) with the empirically accessible physical phenomena in R^3.

Also, the relationship between the two realities, complex and real, evidently possesses some features of *causality* that suggests "physicality". For example, the rotation of an 'invisible' body in the complex space "causes" the Lorentz contraction of its real part as observed in the real space.

Second issue is our use of complex numbers i.e., the C^3 model for the paraphysical space instead of (as some people were trying) the real R^6 version. There are two reasons for that.

First, the complex extension of the real space was not a result of our primary, given in advance, idea. It "happens" as the result of analyzing the real Lorentz transformation as it is described in section 2.

I mean the trick relying on replacing the common algebraic expression $\sqrt{(1 - u^2/c^2)}$ by cosθ and then completing it to cosθ + isinθ, which describes the rotation by □ in the so created complex plane. This fact together with the so obtained very nice "physical" properties inclined me to choose the complex space as the model. One can say, the extension came up in a very natural way, just "by accident". I don't see any such a natural reasoning for the possible R^3 → R^6 extension.

Other reason for the choice is that, generally speaking, the geometry of any C^n (n = 1, 2, …) is much richer and, say, "more rational" (!) than the R^n (as well as R^{2n}) geometry.

Everybody who ever studied an advanced analytic geometry in arbitrary finite dimensions had occasion to find it out. For example, the degree of unification of geometric objects and "mathematical phenomena" is much higher in the complex case.

Even if to compare the cartesian plane R^2 with the complex plane C^1 the amount of interesting and deep mathematical theorems that holds in the later is incomparably higher than whatever holds in R^2.

This richness of properties and high degree of the (logical) unification of complex mathematical models may be considered as the "model's choice motivation". My (and not only my) view is that more rational (in above sense) models more likely are proper as the mathematical models for physics; also from purely methodological viewpoint. If to identify (for a while) 'physical' with "*existence*" let me here cite the short [paraphrased] definition by Descartes: "Existence is a perfection" (see, [13] section 2.1).

According to that, one may say: perfect model (the mathematical form of matter) implies its "embodiment" into perfect (so physical) space, possibly even into the complex projective space P^3 [42] (see the 'Final Remark 1' below).

More rationality and unification actually means more simplicity (here realize, that SR in the complex para-space "becomes" classical Newtonian mechanics) and the overblowing complexity of physics based on real R^3 or M^4 models is the very well-known awkward problem of today. These facts may explain our choice of complex models C^3 over the real, say, R^3 or R^6.

All the considerations from Part C provide a strong indication on some *physicality* of the space modeled by C^3 and the considered objects in it. This *physicality* as being not sensual and not having other physical properties such as direct recognizability by physical instruments I would consider as partial only and the underlying space and the objects call "*paraphysical*". The latter property can be considered as an (based on rational mathematical investigations) extension of the 'physicality'. This extension can still be the subject of, say, "**extended physics**" with the direct and important applications to the (regular) physics.

Final Remark 1: As above mentioned, mathematical properties of C^n are incomparably richer and, as one can say, are more "rational" than properties of the real R^n. What may come to once mind, the Nature (or God?) more likely has chosen, at a possible creation process, the more rational (more "perfect") *possibility* rather than the less. The physicality is, probably, as rational as the mathematics so the CHOICE of a proper form (in the Aristotelian sense) for the physical world (and the underlying space) was an optimal one and therefore was not given to it the form only relying on real numbers (R^3 or M^4 space).

On the other hand, as it also is known from analytic geometry, the geometry of C^n is still not a "perfect geometry". Much more deep properties can be proven when C^n is extended to the **projective complex space**, say, $\mathbf{P^n}$ [42], where points at infinity [the invalid points], each for each particular direction of a straight line (and all other lines parallel to it), are added together with some underlying additional structure. The degree of unification of different concepts and objects (in P^n) is probably the highest possible. For example, all the quadrigas (five types in R^n and two types in C^n) can be reduced to one unit sphere by a kind of linear transformations (the complex projective). Some beautifully constructed objects (for example, the polar hyperplanes their poles and their mutual duality, [42]) and their astonishing properties have no such a place in C^n nor, of course, in R^n. In P^n, many things, finally, become clear and "smooth" achieving their "final perfection".

The natural question that arises is, what would be the physical motivation to place the universe in such a space? At this point recall that the complex Galilean speed of light (corresponding ["topologically"] to its real part, the very well-known **c**) is infinite as it moves parallelly to any imaginary axis [in iR^3]. This infinite speed would then correspond to a point at infinity (in the space of speeds) corresponding to a given light's imaginary direction. These phenomena imply infinite distances (in the space of positions).

Thus, probably, at the end we should arrive at P^3 complex projective para-space for physics.

This is, however, only a suggestion for potential others and is out of scope of this paper.

Final Remark 2: An interpretation of the complex paraphysical space and the objects in it is, to an extent, open. According to my interpretation in [3] the Imaginarity in C^3 stands for (possibly, kinetic) energy. Realize that any increment of the argument θ is equivalent to the increment in speed but also to the increment of the imaginary part of a moving body. One can say R^3 only contains static reality (geometry) but motion arises, together with the complex extension of geometry (R^3), as stimulated by imaginary parts of considered quantities (or, equivalently, by the rotations).

It is, at least partially, a 'physical interpretation' of the imaginary subspace iR^3 only. This interpretation does not exclude the following "mental or psychic interpretation" of a bigger reality $C^3 – R^3$, as given in [10].

It is the very well-known fact that consciousness of any other person (different than any "myself") is not given by senses of a "myself" and, apparently, possesses some ability to act (energy or will). On the other hand, the consciousness has a physical "surface" (brain, body) by which it communicates with other "selves".

The possible **analogy** of the consciousness endowed with a human body and a "Person" [here identified with the whole paraphysical (spiritual?) being whose "body" (an external "surface") is the physical universe, included in R^3] **is striking**.

This strongly resembles the common idea of **God** who, by the way, is essentially different from the "outside" universe and thus is transcendent. Also, according to the complex model, "He" is much "bigger" than the three-dimensional universe. All this is of course the hypothesis, at this stage.

References

1. Schutz BF (1985) A First Course in General Relativity. Cambridge University Press https://assets.cambridge.org/97811084/92676/frontmatter/9781108492676_frontmatter.pdf .
2. Jerzy Filus (2016) A Complex "Time-Free" Model for Both Classical and Relativistic Motion. Academia and ResearchGate https://www.researchgate.net/publication/285589522_A_COMPLEX_TIME_FREE_MODEL_FOR_BOTH_CLASSICAL_AND_RELATIVISTIC_MOTION.
3. Kaiser G (2003) Physical wavelets and their sources: Real physics in complex spacetime. Journal of Physics A: Mathematical and General 36: R291.
4. Newman ET (1973) Maxwell's Equations in Complex Minkowski Space. J Math Phys 14: 202-203.
5. Rauscher E, Targ R (2001) Complex Space-Time Metrics.

Journal of Scientific Exploration 15: 331-354.
6. Rauscher E, Targ R (2006) Investigation of a Complex Space-Time Metric to Describe Precognition of the Future. AIP Conference Proceedings 283: 121-146.
7. Higgs P (1964) Broken Symmetries and the Masses of Gauge Bosons. Physical Review Letters 13: 508-509.
8. Feinberg, Gerald (1967) Possibility of Faster – Than – Light Particles. Physical Review 159: 1089-1105.
9. Bernardi G, Carena M, Junk T (2007) Higgs bosons: Theory and Searches. Review: Hypothetical particles and Concepts. Particle Data Group https://pdg.lbl.gov/2009/reviews/rpp2009-rev-higgs-boson.pdf .
10. Kaiser G (1990) Quantum Physics, Relativity, and Complex Spacetime: Towards a New Synthesis. North-Holland https://www.scirp.org/reference/referencespapers?referenceid=3660771 .
11. Borsuk K (1969) Multidimensional Analytic Geometry. Warszawa: Panstwowe Wydawnictwo Naukowe (Polish Scientific Publishers) Monografie Matematyczne https://library.imar.ro/cgi-bin/koha/opac-detail.pl?biblionumber=16002&query_desc=pl%3A%22Warszawa%20%3A%22.
12. Burkhardt Hans, Smith Barry (1991) Handbook of Metaphysics and Ontology. Philosophia Verlag https://philarchive.org/archive/BURHOM.
13. Van Inwagen Peter, Sullivan Meghan (2014) Metaphysics. Stanford Encyclopedia of Philosophy https://plato.stanford.edu/entries/metaphysics/ .
14. Van Inwagen Peter (2009) Being, Existence, and Ontological Commitment. Oxford University Press 472-506.
15. Hofweber Thomas (2016) Ontology and Ambitions of Metaphysics. The Journal of Philosophy 114: 623-628.
16. Mc Daniel Kris (2009) Ways of Being. Oxford University Press https://philpapers.org/archive/UNKWOB.pdf.
17. Rosen Gideon (1983) Abstract Objects: An Introduction to Axiomatic Metaphysics. Dordrecht: D. Reidel https://mally.stanford.edu/abstracts/book1.html.
18. Fine K (1982) The Problem of Non-Existence. I Internalism. Topoi 1: 97-140.
19. Fine K (1984) Critical Review of Parsons' Nonexistent Objects. Philosophical Studies 45: 94-142.
20. Thomasson A (1999) Fiction and Metaphysics. Cambridge University Press https://www.cambridge.org/us/universitypress/subjects/philosophy/epistemology-and-metaphysics/fiction-and-metaphysics?format=HB&isbn=9780521640800.
21. Reicher M (2016) Nonexistent Objects. The Stanford Encyclopedia of Philosophyhttps://philpapers.org/rec/REINO .
22. Loux M, Zimmerman D (2003) Existence, Ontological Commitment, and Fictional Entities. The Oxford Handbook of Metaphysics, Oxford University Press https://philpapers.org/rec/VANEOC-3 .
23. Yablo Stephen (2009) Must Existence Questions have Answers?. Oxford University Press 507- 525.
24. Berkeley G (1998) A Treatise Concerning the Principles of Human Knowledge. Oxford University Press https://global.oup.com/academic/product/a-treatise-concerning-the-principles-of-human-knowledge-9780198751618?cc=us&l.
25. Hume D (1975) A Treatise of Human Nature. Oxford: Claredon Press https://oll-resources.s3.us-east-2.amazonaws.com/oll3/store/titles/342/0213_Bk.pdf.
26. Hume D (1999) An Enquiry Concerning Human Understanding. Oxford University Press https://fitelson.org/confirmation/hume_enquiry.pdf.
27. Arturo Tozzi (2015) Avenarius R Critique of The Pure Experience. University of North Texas https://www.researchgate.net/publication/273259216_RICHARD_AVENARIUS'_CRITIQUE_OF_THE_PURE_EXPERIENCE_AN_ENGLISH_TRANSLATION_AT_LAST.
28. Mach E (1890) The Analysis of Sensations-Antimeta physical. Monist 1: 48.
29. Mach E (1984) The Analysis of Sensations and the Relation of the Physical to the Psychical. La Salle: Open Court https://ia800906.us.archive.org/19/items/analysisofsensat00mach/analysisofsensat00mach.pdf.
30. Khashaba DR (2002) Subjectivism and Solipsism. Philosophy Pathways https://philpapers.org/rec/KHASAS.
31. Burns K, Gidea M (2005) Differential Geometry and Topology with a View to Dynamical Systems. Taylor and Francis Group LLC https://www.taylorfrancis.com/books/mono/10.1201/9781420057539/differential-geometry-topology-keith-burns-marian-gidea.
32. Jech Thomas (2003) Set Theory. The Third Milenium Edition Revised and ExpandedSpringer https://link.springer.com/book/10.1007/3-540-44761-X.
33. Descartes, Rene (2008) Discourse on Method & Meditation on First Philosophy. Talamh Books https://www.goodreads.com/book/show/31862.Discourse_on_Method_and_Meditations_on_First_Philosophy.
34. Kirk Ludwig (2018) Skepticism, Logical Independence, and Epistemic Priorit. Gainesville https://socrates.sitehost.iu.edu/papers/Skepticism,%20Logical%20Independence,%20and%20Epistemic%20Priority.htm.
35. Klein, Jurgen (2016) Francis Bacon. The Stanford Encyclopedia of Philosophy https://plato.stanford.edu/entries/francis-bacon/.
36. Ahlfors Lars (1979) Complex Analysis (3rd ed). McGraw Hill https://mccuan.math.gatech.edu/courses/6321/lars-ahlfors-complex-analysis-third-edition-mcgraw-hill-science_engineering_math-1979.pdf.
37. Chirka EM (2001) Bergman-Weil Representation. Encyclopedia of Mathematics https://encyclopediaofmath.org/wiki/Bergman-Weil_representation#:~:text=The%20Bergman%E2%80%93Weil%20representation%20defines,than%20that%20of%20%E2%88%82U.
38. Kytmanov AM (1995) The Bochner-Martinelli integral and its applications. Birkhauser Verlag https://link.springer.com/book/10.1007/978-3-0348-9094-6.
39. Rudin W (1973) Functional Analysis. McGraw-Hill https://59clc.wordpress.com/wp-content/uploads/2012/08/functional-analysis-_-rudin-2th.pdf.
40. Vladimirov VS (1979) Generalized Functions in Mathematical Physics.MIR https://archive.org/details/vladimirov-genaralized-equations-in-mathematical-physics.
41. Green MB, Schwarz JH, Witten E (2012) Superstring Theory. Cambridge University Press https://www.cambridge.org/core/books/superstring theory/35152940E2D9C3B0BD327EB7FF74DBBE.
42. Hendricks Jared (2015) Quantum Physics. 3rdEdition Kindle Book https://www.amazon.in/Quantum-Physics-Superstrings-Electrodynamics-Dimensions-ebook/dp/B00WPFOYKA.
43. Jerzy Filus (2016) Complex "Para-Space" Model for Special Relativity Theory. Scholar's Press https://www.researchgate.net/publication/285578753_COMPLEX_EUCLIDEAN_PARASPACE_MODEL_FOR_SR_AS_AN_ALTERNATIVE_FOR_REAL_MINKOWSKI_SPACE-TIME.

44. Kant Immanuel (1783) Prolegomena zu einer jeden Kunftigen Metaphysik. The German edition, https://www.abebooks.com/9781484032145/Prolegomena-k%C3%BCnftigen-Metaphysik-German-Edition-1484032144/plp.