ISSN: 2755-9866

Journal of Health Statistics Reports

SCIENTIFIC Research and Community

Review Article

Open Access

On Inference of Weitzman Overlapping Coefficient in Two Weibull Distributions

Hala Y Maqableh and Omar M Eidous*

Department of Statistics, Yarmouk University Irbid, Jordan

ABSTRACT

Studying overlapping coefficients has recently become of great benefit, especially after its use in goodness-of-fit tests. These coefficients are defined as the amount of similarity between two statistical distributions. This research examines the estimation of one of these overlapping coefficients, which is the Weitzman coefficient Δ, assuming two Weibull distributions and without using any restrictions on the parameters of these distributions. We studied the relative bias and relative mean square error of the resulting estimator by implementing a simulation study. The results show the importance of the resulting estimator.

***Corresponding author**

Omar M Eidous, Department of Statistics, Yarmouk University Irbid, Jordan.

Received: July 30, 2024; **Accepted:** August 20, 2024; **Published:** September 05, 2024

Keywords: Weitzman Overlapping Coefficient, Maximum Likelihood Method, Parametric Method, Weibull Distribution, Relative Bias, Relative Mean Square Error

Introduction

Overlapping coefficients are used to determine similarity between two populations. If we want to make comparative inferences about two populations, we will look at what are called measures of similarity or measures of dissimilarity. In fact, when we discuss this type of measure, we should pay attention to the overlapping (OVL) coefficients. Therefore, OVL coefficients are measure of agreement or similarity between two probability statistical distributions.

There are five main measures of OVL, which are: Matusita (ρ), Morisita (λ), Pianka (PI), Kullback –Leibler (KL) and Weitzman (Δ) coefficients. They are defined as follows:

Assuming $f_1(x)$ and $f_2(x)$ are continuous probability density functions, the five OVL coefficients are:

1. Matusita coefficient (1955):

$$\rho = \int \sqrt{f_1(x) f_2(x)}\, dx$$

2. Morsita coefficient (1959):

$$\lambda = \frac{2\int f_1(x) f_2(x)\, dx}{\int [f_1(x)]^2\, dx + [f_2(x)]^2\, dx}$$

3. Pianka coefficient (Chaubey et al., 2008):

$$PI = \frac{\int f_1(x) f_2(x)\, dx}{\sqrt{\int [f_1(x)]^2 dx \int [f_2(x)]^2 dx}}$$

4. Kullback-Leibler coefficient (Dhaker et al., 2019)

$$KL = \frac{1}{1 + \int \big(f_1(x) - f_2(x)\big) log\big(f_1(x)/f_2(x)\big)\, dx}$$

5. Weitzman coefficient (1970):

$$\Delta = \int min\,\{\, f_1(x), f_2(x)\,\} dx$$

The Weitzman coefficient Δ is a widely used and more clearly defined than the other coefficients, which represents the area of intersection between two probability density functions [1-3]. Our interest in this paper is only the Weitzman coefficient Δ. The main objective is to estimate Δ assuming two Weibull distributions and without using any restrictions on the parameters of these distributions. If the value of any of the above five coefficients is 1 (i.e. OVL=1) then $f_1(x) = f_2(x)$. If OVL=0, then the supports of the two densities $f_1(x)$ and $f_2(x)$ have no interior points.

Overlap measures are applied in different areas like, genetic ecology income reliability analysis and goodness of fit test [4-9].

Weibull Distribution and OVL Coefficients

The Weibull distribution is a continuous probability density function. This distribution attracted the interest of statisticians due to its advantages such as its flexibility to model data sets in many fields of applied statistics, like, lifetime data, economics and business administration data, engineering studies data and wind power data [10,11]. Let X be a continuous random variable that follows a Weibull distribution with a scale parameter α and a shape parameter β then the pdf of X is,

$$f(x) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} e^{-(x/\alpha)^{\beta}}, \qquad x > 0, \qquad \alpha, \beta > 0.$$

This will be denoted by $X \sim We(\alpha,\beta)$. Some well-known statistical distributions are special cases of $We(\alpha,\beta)$, including the exponential distribution, which is obtained if β=1, and the Rayleigh distribution, which is obtained if $\beta = 2$ and $\alpha=\sqrt{2}\sigma$ [30]. Let $X \sim W(\alpha_1,\beta_1)$ and $Y \sim W(\alpha_2, \beta_2)$ where X and Y are independent random variables. Assume that $\beta_1 = \beta_2 = 1$, the OVL coefficients λ, ρ and Δ were studied by who also studied the effect of sampling plan on these OVL coefficients. Under the assumption, $\beta_1 = \beta_2 = \beta$, the coefficients λ, ρ and Δ were studied by while the coefficients *PI* and *KL* were studied by [12-15]. Finally, without using any assumptions on the parameters of Weibull distributions, were concerned with the coefficients λ and ρ, while the study by Eidous and focused on the coefficient Δ. In the last both studies, the numerical integration approximation method was used to study the various coefficients [16,17].

There are researches that have studied OVL coefficients under statistical distributions other than Weibull distributions. considered the case of normal distributions [18-21].

Helu and investigated the OVL coefficients of Lomax distributions with different sampling procedures. Parametric methods for estimating the confidence interval for Δ have been studied by Wang and who also proposed methods for estimating the confidence interval for Δ undera variety of distributions, including the normal distribution [22,23].

There are also some nonparametric studies that were concerned with studying OVL coefficients, which can be found in the literature. These studies do not assume any specific statistical distributions for the phenomenon under study. See for example, Schmid and [24-28].

Main Results

Let $X_1, X_2, \ldots, X_{n1}$ be a random sample from $W(\alpha_1,\beta_1)$, and let $Y_1, Y_2, \ldots, Y_{n2}$ be another random sample from $W(\alpha_2,\beta_2)$, where the two samples are independent. If $\beta_1 = \beta_2 = \beta$, let $\hat{\alpha}_1, \hat{\alpha}_2$, and $\hat{\beta}$ are the maximum likelihood estimators (MLEs) of α, α, β respectively. If there is no restriction about the distribution's parameters, let $\hat{\alpha}_1$, $\hat{\alpha}_2$, $\hat{\beta}_1$ and $\hat{\beta}_2$ are the MLEs of $\alpha_1, \alpha_2, \beta_1$ and β_2 [29].

To estimate the Weitzman Coefficient Δ based on these two random samples, we first express the coefficient Δ as follows:

To simplify the notations, let $f_1(X) = W(\alpha_1,\beta_1)$ and $f_2(X) = W(\alpha_2, \beta_2)$. Consider $\min\{f_1(X), f_2(X)\}/ f_1(X)$ as a function of X and $\min\{f_1(Y), f_2(Y)\}/ f_2(Y)$ as a function of Y. Now,

$$\begin{aligned} E\left(\frac{min\{f_1(X), f_2(X)\}}{f_1(X)}\right) &= \int_0^{\infty} \frac{min\{f_1(x), f_2(x)\}}{f_1(x)} f_1(x)\, dx \\ &= \int_0^{\infty} min\{f_1(x), f_2(x)\} dx \\ &= \Delta \end{aligned}$$

and

$$\begin{aligned} E\left(\frac{min\{f_1(Y), f_2(Y)\}}{f_2(Y)}\right) &= \int_0^{\infty} \frac{min\{f_1(y), f_2(y)\}}{f_2(y)} f_2(y)\, dy \\ &= \int_0^{\infty} min\{f_1(y), f_2(y)\}\, dy \\ &= \int_0^{\infty} min\{f_1(x), f_2(x)\} dx \\ &= \Delta. \end{aligned}$$

Also, Δ can be expressed as follows,

$$\Delta = \frac{1}{2}\left[E\left(\frac{min\{f_1(X), f_2(X)\}}{f_1(X)}\right) + E\left(\frac{min\{f_1(Y), f_2(Y)\}}{f_2(Y)}\right)\right]$$

If $\hat{f}_1(X) = W(\hat{\alpha}_1, \hat{\beta}_1)$ and $\hat{f}_2(X) = W(\hat{\alpha}_2, \hat{\beta}_2)$ then

$$\Delta = E\left(min\{f_X(X), f_Y(X)\}/ f_X(X)\right)$$

can be estimated by using the method of moments as given below,

$$\hat{\Delta} = \frac{1}{n_1}\sum_{k=1}^{n_1}\left(\frac{min\{\hat{f}_1(X_k), \hat{f}_2(X_k)\}}{\hat{f}_1(X_k)}\right)$$

Also, $\Delta = E\left(min\{f_X(Y), f_Y(Y)\}/ f_Y(Y)\right)$ can be estimated by,

$$\hat{\Delta} = \frac{1}{n_2}\sum_{k=1}^{n_2}\left(\frac{min\{\hat{f}_1(Y_k), \hat{f}_2(Y_k)\}}{\hat{f}_2(Y_k)}\right).$$

The average of the last two estimators can be considered as the third estimator for Δ, which is given by,

$$\hat{\Delta} = \frac{1}{2n_1}\sum_{k=1}^{n_1}\left(\frac{min\{\hat{f}_1(X_k), \hat{f}_2(X_k)\}}{\hat{f}_1(X_k)}\right) + \frac{1}{2n_2}\sum_{k=1}^{n_2}\left(\frac{min\{\hat{f}_1(Y_k), \hat{f}_2(Y_k)\}}{\hat{f}_2(Y_k)}\right).$$

Note that if we assume that $\beta_1 = \beta_2 = \beta$ then $\hat{f}_1(X) = W(\hat{\alpha}_1, \hat{\beta})$ and $\hat{f}_2(X) = W(\hat{\alpha}_2, \hat{\beta})$. The performances of these three estimators are investigated in a preliminary simulation study. The results show that the performance of $\hat{\Delta}$ (last version) is more stable than the first two versions.

Simulations

A simulation study is conducted to compare the performances of the proposed estimator Δ^(last version in the previous section) of Δ with some existing counterparts that developed in the literature. In particular, the nonparametric kernel estimator is considered for this purpose, which is denoted by $\hat{\Delta}_k$. It is important to note that the kernel estimator is a general estimator, which does not require any assumptions about the shape of the underlying sample distribution

To cover most possible cases in practical applications, the two independent samples $x_1, x_2, \ldots, xn_1$ and $y_1, y_2, \ldots, y\, n_2$ are simulated from 12 pairs of Weibull distributions. From these pairs, four pairs with the same scale parameters (i.e. $\alpha_1 = \alpha_2$), four pairs with the same shape parameters (i.e. $\beta_1 = \beta_2$) and four pairs with different scale parameters and different shape parameters were selected. Although these choices seem arbitrary, the goal was to allow the exact overlapping coefficient values to vary from small (near 0) to large (near 1). In Tables (1)-(3), the parameter values are shown along with the exact value of Δ for each pair. To study the effect of sample sizes on the behavior of each estimator, for following ample sizes were taken (n_1, n_2) = (10,10),(20,30),(30,30),(50,50),(100,200).

Numerical results were calculated based on a thousand iterations (R=1000) using Mathematica, Version 7. For each estimator, we calculated the Relative Bias (RB), Relative Root Mean Square Error (RRMSE) and Efficiency (EFF), which can be defined as follows,

$$RB = \frac{\hat{E}(estimator) - exact}{exact},$$

$$RRMSE = \frac{\sqrt{\widehat{MSE}(estimator)}}{exact}$$

and

$$EFR = \frac{\widehat{MSE}(kernel)}{\widehat{MSE}(proposed\ estimator)}.$$

Note that if Δ is the estimator of $\hat{\Delta}$ and if $\hat{\Delta}_{(j)}$ is the value of $\hat{\Delta}$ computed based on a sample of iteration j, j=1,2,…,R=1000 then

$$\hat{E}(\hat{\Delta}) = \sum_{j=1}^{R} \hat{\Delta}_{(j)} / R$$

and

$$\widehat{MSE}(\hat{\Delta}) = \sum_{j=1}^{R} \left(\hat{\Delta}_{(j)} - E(\hat{\Delta})\right)^2 / R.$$

Note also that the kernel estimator requires specifying two quantities, the first is the kernel function, which took the standard normal function, and the other quantity is the smoothing parameter, which is calculated using the rule,

$$1.06\, S\, n^{-1/5}$$

where S is the usual standard deviation for the interested sample. It is worth noting here that there are other ways to calculate the smoothing parameter [30]. However, we found that the performance of the kernel estimator using the above rule is very acceptable.

Simulation Results

All computations and outputs of the simulation study are presented in Tables (1-3). From these simulation results, we can conclude the following:

It is obvious that |RB|s that are associated with the kernel estimator Δ^_k are large compared with other the proposed estimator, especially for small samples sizes. Most RBs values of the kernel estimates are negative, which indicates that -on the average- the kernel estimates underestimate the true value of the corresponding coefficient. It appears tl$\hat{\Delta}$t the problem of underestimate is a problem associated with the kernel estimates, even in other fields such as line transect method and degradation methods [31-35].
$\hat{\Delta}_k$

The values |RB|s of the different proposed estimate are much smaller than that of the kernel estimate for al$\hat{\Delta}_k$ɔst all considered cases.

As the samples sizes increase the RRMSE of the two ‹$\hat{\Delta}_k$imato$\hat{\Delta}$ decrease. This is a good sign for the consistency of the estimators that considered in this study.

The values of RRMSEs and consequently the values of EFFs for the proposed estimate for different cases indicate that it performs better than the kernel estimate.

The proposed estimator Δ performs very well even when the data are simulated from pair Weibull distributions with equal scale or with equal shape parameter. By taking into account that Δ is developed without any assumption on the shape of pair distributions, its performance is acceptable but not as that of the proposed one Δ^. However, we expect that Δ may be perform better than Δ^if the underlying data distribution is not Weibull.

Table 1: The RB, RRMSE and EFF of the estimators $\hat{\Delta}_k$, and $\hat{\Delta}$ when the data are simulated from pair Weibull distributions with equal scale parameters ($\alpha_1 = \alpha_2 = 1$)

		$\Delta_{exact} = 0.8678$ (β_1, β_2) = (3,4)		$\Delta_{exact} = 0.6774$ (β_1, β_2) = (3,6.2)	
($n1$,$n2$)		$\hat{\Delta}_k$	$\hat{\Delta}$	$\hat{\Delta}_k$	$\hat{\Delta}$
	RB	-0.2744	-0.0608	-0.3009	0.0184
	RRMSE	0.2967	0.1271	0.3190	0.1228
	EFF	**1.0000**	**2.3346**	**1.0000**	**2.5976**
	RB	-0.2016	-0.0319	-0.2018	-0.0249
	RRMSE	0.2160	0.0753	0.2407	0.1172
	EFF	**1.0000**	**2.8661**	**1.0000**	**2.0543**
	RB	-0.1086	0.0006	-0.0560	0.0139
	RRMSE	0.1122	0.0503	0.0827	0.0608
	EFF	**1.0000**	**2.2285**	**1.0000**	**1.3587**

		Δ_{exact} = 0.4880 (β_1, β_2) = (3, 10.3)		Δ_{exact} = 0.2979 (β_1,β_2) = (3,20.4)	
(20, 30)	RB	-0.4367	-0.0795	-0.5952	-0.0374
	RRMSE	0.4609	0.1449	0.6324	0.2506
	EFF	**1.0000**	**3.1809**	**1.0000**	**2.5232**
(50, 50)	RB	-0.3203	-0.0366	-0.5012	-0.0488
	RRMSE	0.3418	0.1060	0.5250	0.1264
	EFF	**1.0000**	**3.2245**	**1.0000**	**4.1518**
(100,200)	RB	-0.0707	0.0138	-0.1991	0.0103
	RRMSE	0.1084	0.0796	0.2314	0.0824
	EFF	**1.0000**	**1.3620**	**1.0000**	**2.8073**

Table 2: The RB, RRMSE and EFF of the estimators $\hat{\Delta}_k$ and $\hat{\Delta}$ when the data are simulated from pair Weibull distributions with equal scale parameters ($\beta_1 = \beta_2 = 3$).

		Δ_{exact} = 0.8012 (α_1,α_2) = (1,1.2)		**Δ *exact* = 0.5783 (α_1,α_2)=(1,1.5)**	
$(n1,n2)$		$\hat{\Delta}_k$	$\hat{\Delta}$	$\hat{\Delta}_k$	$\hat{\Delta}$
(20, 30)	RB	-0.2269	-0.0538	-0.2052	-0.0894
	RRMSE	0.2526	0.1264	0.2494	0.1540
	EFF	**1.0000**	**1.9990**	**1.0000**	**1.6192**
(50, 50)	RB	-0.1382	-0.0449	-0.0874	-0.0301
	RRMSE	0.1622	0.1192	0.1543	0.1156
	EFF	**1.0000**	**1.3610**	**1.0000**	**1.3355**
(100,200)	RB	-0.0723	-0.0109	-0.0420	-0.0201
	RRMSE	0.0893	0.0526	0.0956	0.0815
	EFF	**1.0000**	**1.6964**	**1.0000**	**1.1737**
(20, 30)	RB	-0.1293	-0.0021	-0.2503	-0.0477
	RRMSE	0.2810	0.2494	0.5722	0.5193
	EFF	**1.0000**	**1.1266**	**1.0000**	**1.1020**
(50, 50)	RB	-0.0762	-0.0127	-0.1459	0.0125
	RRMSE	0.2004	0.1571	0.4373	0.3976
	EFF	**1.0000**	**1.2748**	**1.0000**	**1.0998**
(100,200)	RB	-0.0428	-0.0043	-0.0989	-0.0461
	RRMSE	0.1013	0.0875	0.3231	0.2938
	EFF	**1.0000**	**1.1576**	**1.0000**	**1.0997**

Table 3: The RB, RRMSE and EFF of the three estimators $\hat{\Delta}_k$, and $\hat{\Delta}$ when the data are simulated from pair Weibull distributions with different scale and different shape parameters

		Δ_{exact}=0.8672 (α_1, α_2) = (1,1.2) (β_1, β_2) = (2,1.8)		**Δ_{exact} = 0.6243 (α_1,α_2)=(1,1.5) (β_1, β_2) = (3,1.9)**	
(n_1, n_2)					
(20, 30)	RB	-0.2387	-0.1015	-0.1796	-0.0220
	RRMSE	0.2728	0.1694	0.2498	0.1408
	EFF	**1.0000**	**1.6103**	**1.0000**	**1.7732**
(50, 50)	RB	-0.0925	-0.0006	-0.1167	-0.0353
	RRMSE	0.1157	0.0673	0.1497	0.1136
	EFF	**1.0000**	**1.7187**	**1.0000**	**1.3176**
(100,200)	RB	-0.0450	-0.0120	-0.0617	-0.0254
	RRMSE	0.0644	0.0583	0.0898	0.0658
	EFF	**1.0000**	**1.1049**	**1.0000**	**1.3653**

		Δ_{exact}=0.4370 $(\alpha_1, \alpha 2)$ = (1,1.8) (β_1, β_2) = (4,2.1)		Δ_{exact} = 1646 (α_1,α_2)=(1,3) (β_1,β_2)=(6,2)	
(20, 30)	RB	-0.2465	-0.0254	-0.3618	0.0263
	RRMSE	0.3257	0.1981	0.4706	0.3359
	EFF	**1.0000**	**1.6443**	**1.0000**	**1.4007**
(50, 50)	RB	-0.1510	-0.0271	-0.2848	0.0067
	RRMSE	0.2250	0.1668	0.3588	0.2153
	EFF	**1.0000**	**1.3488**	**1.0000**	**1.6659**
(100,200)	RB	-0.0779	-0.0064	-0.2040	-0.0339
	RRMSE	0.1234	0.0802	0.2339	0.1239
	EFF	**1.0000**	**1.5384**	**1.0000**	**1.8881**

References

1. Matusita K (1955) Decision rules, based on the distance, for problems of fit, two samples, and estimation. The Annals of Mathematical Statistics 26: 631-640.
2. Morisita M (1959) Measuring of interspecific association and similarity between communities. Mem Fac Sci Kyushu Univ Series E 3: 65-80.
3. Chaubey G, Metspalu M, Karmin M, Thangaraj K (2008) Language shift by indigenous population: A model genetic study in South Asia. International Journal of Human Genetics 8: 41.
4. Federer WT, Powers L, Payne MG (1963) Studies on statistical procedures applied to chemical genetic data from sugar beets. Technical Bulletin, Agricultural Experimentation Station, Colorado State University https://books.google.co.in/books/about/Studies_on_Statistical_Procedures_Applie.html?id=NwFQAQAAMAAJ&redir_esc=y.
5. Pianka ER (1973) The structure of lizard communities. Annual Review of Ecology and Systematics 4: 53-74.
6. Gastwirth JL (1975) Statistical measures of earnings differentials. The American Statistician 29: 32-35.
7. Ichikawa M (1993) A meaning of the overlap pedarea under probability density curves of stress and strength. Reliability Engineering and System Safety 41: 203-204.
8. Dhaker H, Ngom P, Mbodj M (2019) Overlap coe-cients based on Kullback-Leibler divergence: exponential populations case. International Journal of Applied Mathematical Research 6: 135-140.
9. Alodat T, Al Fayez M, Eidous O (2022) On the asymptotic distribution of Matusita's overlapping measure. Communications in Statistics - Theory and Methods 51: 6963-6977.
10. Rinne H (2008) The Weibull distribution: A handbook. CRC Press, Taylor & Francis Group, New York https://www.taylorfrancis.com/books/mono/10.1201/9781420087444/weibull-distribution-horst-rinne.
11. Wais P (2017) A review of Weibull functions in wind sector. Renewable and Sustainable Energy Reviews 70: 1099-1107.
12. Madhuri SM, Sherry G, Subhash A (2001) Estimating overlap of two exponential populations. Proceedings of the Annual Meeting of the American Statistical Association 2: 848-851.
13. Samawi HM, Al-Saleh MF (2008) Inference of overlapping coefficients in two exponential populations using ranked set sample. Communication of Korean of Statistical Society 15: 147-159.
14. Al-Saidy O, Samawi HM, Al-Saleh MF (2005) Inference on overlapping coefficients under the Weibull distribution: Equal shape parameter. ESAIM: Probability and Statistics 9: 206-219.
15. Eidous O, Abu Al-Hayja'a M (2023) Weibull distributions for estimation of Pianka and Kullback-Leibler overlapping measures. Journal of Mathematics and Statistics Research 5: 165.
16. Eidous OM, Abu Al-Hayjaa M (2023) Estimation of overlapping measures using numerical approximations: Weibull distributions. Jordan Journal of Mathematics and Statistics (JJMS) 16: 741-761.
17. Eidous O, Abu Al-Hayja's M (2023) Numerical integration approximations to estimate the Weitzman overlapping measure: Weibull distributions. Yugoslav Journal of Operations Research 33: 699-712.
18. Inman HF, Bradley EL (1989) The overlapping coefficient as a measure of agreement between probability distributions and point estimation of the overlap of two normal densities. Communication in Statistics- Theory and Methods 18: 3851-3874.
19. Mulekar MS, Mishra SN (1994) Overlap coefficients of two normal densities: equal means case. Journal of the Japan Statistical Society 24: 169-180.
20. Eidous O, Al-Daradkeh S (2022) Estimation of Matusita overlapping coefficient for pair normal distributions. Jordan Journal of Mathematics and Statistics (JJMS) 15: 1137-1151.
21. Eidous OM, Al-Shourman A (2023) Estimating the Weitzman overlapping coefficient using integral approximation method in the case of normal distributions. Applied Mathematics, Modeling and Computer Simulation 42: 1011-1020.
22. Helu A, Samawi H, Vogel R (2011) Nonparametric overlap coefficient estimation using ranked set sampling. Journal of Nonparametric Statistics 23: 385-397.
23. Wang D, Tiana L (2017) Parametric methods for confidence interval estimation of overlap coefficients. Computational Statistics & Data Analysis 106: 12-26.
24. Schmid F, Schmidt A (2005) Nonparametric estimation of the coefficient of overlapping-theory and empirical application. Computational Statistics & Data Analysis 50: 1583-1596.
25. Pastore M (2018) Overlapping: R package for estimating overlapping in empirical distributions. Journal of Open Source Software 3.
26. Eidous OM, AL-Talafha SA (2022) Kernel method for overlapping coefficients estimation. Communications in Statistics: Simulation and Computation 51: 5139-5156.
27. Eidous OM, Ananbeh EA (2024) Kernel method for estimating overlapping coefficient using numerical integration methods. Applied Mathematics and Computation 462: 128339.
28. Eidous OM, Ananbeh EA (2024) Kernel method for estimating Matusita overlapping coefficient using numerical approximations. Annals of Data Science https://doi.org/10.1007/s40745-024-00563-y.

29. Eidous OM, Marie M, Al-Haj Ibrahim M (2010) A comparative study for bandwidth selection in kernel density estimation. Journal of Modern Applied Statistical Methods 9: 263-273.
30. Eidous O (2009) Kernel method starting with half-normal detection function for Line transect density estimation. Communications in Statistics-Theory and Methods 38: 2366-2378.
31. Eidous O (2011) Variable location kernel method using line transect sampling. Environmetrics 22: 431-440.
32. Eidous O (2012) A new kernel estimator for abundance using line transect sampling without the shoulder condition. Journal of the Korean Statistical Society 41: 267-275.
33. Ba Dakhn LN, Al-Haj Ebrahem M, Eidous O (2017) Semi-parametric method to estimate the time-to-failure distribution and its percentiles for simple linear degradation model. Journal of Modern Applied Statistical Methods 16: 322-346.
34. Arif OH, Eidous O (2017) Fourth-order kernel method for simple linear degradation model. Communications in Statistics-Simulation and Computation 47: 16-29.
35. Weitzman MS (1970) Measures of overlap of income distributions of white and Negro families in the United States. US Bureau of the Census 22.