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On Resolvability of Graphs Associated with Vector Spaces

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ABSTRACT

Let $\mathbb{V}$ be a n-dimensional vector space over the field F with a basis $\mathfrak{B}=\{\alpha_1,\alpha_2,\ldots,\alpha_n\}$. In this paper, we obtain the resolving parameters like metric dimension and partition dimension of graphs associated with vector space. Also, found the values of metric-locating-domination number, locating-domination number, and bipartite decomposition of kind of graph associated with vector space.

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Introduction

Let $\mathbb{F}$ be a finite dimensional vector space over the field $\mathbb{F}$ with with $\mathfrak{B}=\alpha_1,\alpha_2,\ldots,\alpha_n$ as a basis. Any vector $v\in\mathbb{V}$ can be expressed uniquely as a linear combination $v=a_1\alpha_1+a_2\alpha_2+\ldots+a_n\alpha_n$ where $a_i\in\mathbb{F}$ and the same is denoted by $v=(a_1,a_2,\ldots a_n)$. The skeleton of $v\in\mathbb{V}\setminus\{0\}$ with respect to $\mathfrak{B}$ is defined as $S_{\mathfrak{B}}(v)=\{\alpha_i : a_i\neq 0, i=1,2,\ldots,n\}$

The non-zero component union graph $\Gamma(\mathbb{V}_{\mathfrak{B}})$ of $\mathbb{V}$ respect to $\mathfrak{B}$ is the simple graph with vertex set $V=\mathbb{V}\setminus 0$ and two distinct non-zero vectors $u,v\in V$ are adjacent if and only if $S_{\mathfrak{B}}(u)\cup S_{\mathfrak{B}}(v)=\mathfrak{B}$ g. This graph is introduced and studied by A Das in [1], in [2] the author's found the Topological indices of non-zero component union graph and in [3] The author's produce the result related to the genus of non-zero component union graphs of vector spaces. Also, he constructed nonzero component graph of finite dimensional vector space in [4] and resolving properties of nonzero component graph are obtained by U Ali in [5].

S Maity and AK Bhuniya was defined and studied the linear dependent graph of vector space, whose vertex set is $\mathbb{V}$ and edge set is defined as two vertices are adjacent if and only if they are linearly dependent [6]. The linear dependent graph of vector space is denoted by $\Gamma(\mathbb{V})$.In [6], the completeness, diameter, independent number, clique number, chromatic, Eulerian, vertex connectivity and edge connectivity of linear dependent graph of vector space are studied.

A graph $G=(V,E)$ be a simple graph with non-empty vertex set V and edge set E. The number of elements in V is called order of G and the number of elements in E is called the size of G. A graph G is said to be complete if any pair of distinct vertices is adjacent in G. we denote the complete graph of order n by K_n. One point union of n copies of a graph G is defined as all the vertices in n copies of graph G is adjacent to new vertex and it is denoted by G^n A graph G is bipartite if the vertex V can be partitioned into two disjoint subsets with no pair of vertices in one subset is adjacent. A star graph is a bipartite graph with any one of the partitions containing a single vertex and the same is called as the center of the star graph. A graph G is connected if there exists a path between every pair of distinct vertices in G. The degree of the vertex $v\in V$, denoted by $d(v)$,is the number of edges in G which are incident with v A graph G is said to be r-regular if the degree of all the vertices in G is r. The diameter of a connected graph is supreme of shortest distance between vertices in G and is denoted by $diam$ (G). The girth of G is defined as length of the shortest cycle in G and is denoted by $gr(G)$ If G contains no cycles then, $gr(G)=\infty$. A walk in a graph G is a finite non-null sequence

$W=v_0e_1v_1e_2\ldots e_kv_k$, whose terms are alternatively vertices and edges, such that, for $1\leq i\leq k$ and ends of e_i are v_i–1 and v_i . The walk W is said to be a trial if the edges e_1,..., e_k of the walk W are distinct. Further if vertices $v_0,v_1,\ldots,v_k$ are also distinct, then W is called a path. The distance between two vertices $u,v\in V$ is the length of a shortest path between them and it is denoted by $d(u,v)$ Given a vertex u in a graph G, the open neighborhood of u in G is the set $\{v\in V \mid d(u,v)=1\}$ and it is denoted by $N(u)$. The closed neighborhood of u in G denoted by $N[u]$ is the set $\{v\in V \mid d(u,v)=1\}\cup u$. For two vertices u and v in a graph G, denoted by N[u] is the set $\{v\in V \mid d(u,v)=1\}\cup u$. For two vertices u and v in a graph G, denoted by $N[u]$ is the set $\{v\in V \mid d(u,v)=1\}\cup u$. For two vertices u and v in a graph G, define $u\equiv v$ if $N[u]=N[v]$ or $N[u]=N[v]$. Equivalently, $u\equiv v$ if and only if $N(u)\setminus\{u\}=N(v)\setminus\{u\}$. The relation $\equiv$ is an equivalence relation (see[10]). If $u\equiv v$, then u and v are called twins. The set of vertices is called a twin-set if any two of its vertices are twins.

A set $W\subset V$ is a resolving set if for each pair of distinct vertices $u,v\in V$ there is a vertex $w\in W$ such that $d(u,w)\neq d(v,w)$. A resolving set containing a minimum number of vertices is called a minimum resolving set or a basis for G. The cardinality of a minimum resolving set is called the resolving number or dimension of G and is denoted by *diam* (G). A resolving set W is said to be a star resolving set if it induces a star, and a path resolving set if it induces a path. The minimum cardinality of these sets is called the star resolving number and path resolving number its denoted respectively by $sr(G)$ and $pr(G)$ A subset $T\in V$ and a vertex v of G, the distance $d(v, T)$ between v and T is defined as $d(v,T)=min\{d(v,x)\mid x\in T\}$. For an ordered k-partition $\Pi=\{T_1,T_2,\ldots,T_K\}$ of V and a vertex $v\in V$, the representation of v with respect to Π is defined as k-vectors $r(v\mid\Pi)=(d(v,T_1),d(v,T_2),\ldots,d(v,T_k))$. The partition Π is called a resolving partition if the k-vectors $r(v\setminus\Pi), v\in V,$ are distinct. The minimum k for which there is a resolving k-partition of V is the partition dimension $pd(G)$ of G.

A subset D of V is called dominating set if any vertex in $V \setminus D$ is adjacent with at least one vertex in D. The minimum cardinality of D is called domination number and it is denoted by $\gamma(G)$ The observation rules are as follows

1. Any vertex that is incident to an edge is observed.
2. Any edge joining two vertices is observed.
3. If a vertex is incident to a total of $k > 1$ edges and if $k - 1$ of these edges are observed

Then all k of these edges is observed. A set S to be a power dominating set of a graph if every vertex and every edge in the system is observed by the set S. The power domination number $\gamma_p(G)$ of a graph G is the minimum cardinality of a power dominating set of graphs G. A set of vertices of G is called a metric-locating-dominating set for G if it is resolving and dominating. The metric-locating-dominating number, denoted by mld_G, is the minimum cardinality of a metric-locating-dominating set of G. a metric-location-dominating set L is called locating-dominating set if $N(u)\cap L\neq N(v)\cap L$ for every two vertices $v,u\in V\setminus L$. The locating-domination number, denoted by ldG , is

the minimum cardinality of a locating-dominating set of G. a graph G is said to be embedded in a surface S if G can be drawn in S such that edges intersect only at vertices of G. The genus of graph G is denoted by g (G), is the minimum integer n such that the graph can be embedded in S_n, where S_n denotes the sphere with n handles. For undefined terms in graph theory, we refer [7].

We list out certain existing results which will be referred in this paper.

Theorem 1.1
([1, Theorem 4.2]) Let $\mathbb{V}$ be an n – dimensional vector space over a finite field $\mathbb{F}$ with q elements. Then $\Gamma(\mathbb{V}_{\mathfrak{B}})$ is complete if and only if $\mathbb{V}$ is one-dimensional and $|\mathbb{F}|=2$.

Lemma 1.2
([8, PP. 341]) Suppose u, v are twins in a connected graph Γ and W resolves Γ. Then, u or v is in W. Moreover, if $u\in W$ and $v\notin W$, then $(W\setminus\{u\})\cup\{v\}$ also resolves Γ.

Lemma 1.3
([9, Lemma 2.2.]) Let Π be resolving partition of the vertex set V and $u,v\in V.\ if\ d(u,w)=d(v,w)$ for all $v\in V\{u,v\}$, then u and v belong to different classes of Π.

Theorem 1.4
([6, Theorem 2.4]) The domination number of $\Gamma(\mathbb{V})$ is 1.

Lemma 1.5
([10, Theorem 6.38]) $g(k_n)=\left\lceil\frac{(n-3)(n-4)}{12}\right\rceil$ if n ≥ 3. In particular $g(K_n)=1$ if n= 5,6,7.

Lemma 1.6
([11, Lemma 1]) if G_1, G_2 and G are connected graphs such that $G=G_1\cup G_2=v\,(a\ vertex\ of\ G)$, then $g(G)\leqq g(G_1)+g(G_2)$

Lemma 1.7
([11, Lemma 2]) if is a connected graph having a subgraph G_1 and a block G_2 such that $G=G_1\cup G_2=V\,(a\ vertex\ of\ G)$, then $g(G)\geqq g(G_1)+g(G_2)$.

Metric Dimension of $\Gamma(\mathbb{V}_B)$
In this section, we found the values of resolving number, metric-locating-domination number, locating-domination number of non-zero component union graph vector space $\Gamma(\mathbb{V}_{\mathfrak{B}})$

Lemma 2.1: *The equivalence relations $\equiv$ are $\cong$ same in* $\Gamma(\mathbb{V}_{\mathfrak{B}})$.

Proof
Let $u,v\in\mathbb{V}_{\mathfrak{B}}$ with $u\cong v$. Then, $S_u = S_v$. Hence any vertex w is adjacent to u in $\Gamma(\mathbb{V}_{\mathfrak{B}})$ if and only if w is adjacent to v. Since, $S_u\cup S_w=S_v\cup S_w=\mathfrak{B}$. Conversely, Let $u\equiv v$. Then, $N(u)\setminus\{v\}=N(v)\setminus\{u\}$. We have to show that Suppose, we assume that $S_u\neq S_v$, exit some $\alpha_i\in S_u$ with $\alpha_i\in S_v$. Now consider the vertex w with $S_w=\mathfrak{B}\setminus S_u$. *If* $w=v$ consider the vertex w with S_w is adjacent to u but not v. Otherwise, w is adjacent to u but not v. The above two cases we get contradiction. Hence $S_u = S_v$.

Lemma 2.2
Let $\mathbb{V}$ be an $n\geq 2$ dimensional vector space over the field $\mathbb{F}$ of order 2 with the basis $\mathfrak{B}=\{\alpha_1,\alpha_2,\ldots,\alpha_n\}$ and $X=\{v_i:1\leq i\leq n\}$ where $S_{(v_i)}=\mathfrak{B}\setminus\{\alpha_i\}$. Then $X\setminus\{v_k\}$ is contained in any resolving set of $\Gamma(\mathbb{V}_{\mathfrak{B}})$. Also $X\setminus\{v_k\}$ is not a resolving set.

Proof
For q = 2, a twin-set C_v is of length 1 for every element of $\mathbb{V}$ Let W is resolving set of $\Gamma(\quad)$ $X=\{v\quad\leq i\leq n\}$ where $S_{(vi)}=\mathfrak{B}\setminus\{\alpha_i\}$ *and* v be the vertex whose skeleton is $S_v=\alpha_1,\alpha_2,\ldots,\alpha_n$, i.e.$v=\alpha_1+\alpha_2+\ldots+\alpha_n$. Suppose any two elements v_i and v_j in X is not in W. Then (α_i, α_j) has no resolving element in W. Since, (α_i, α_j) is resolved by either v_i or v_j and $N(\alpha_i) = v_i$. Which is a contradiction. Hence $X\setminus\{v_k\}$ is contained in any resolving set. Also $X\setminus\{v_k\}$ is not a resolving set, since there is no resolving element of $(v,v_k)\,inX\setminus\{v_k\}$ for some $v_k\in X$

Theorem 2.3
Let n,q be integers. Let $\mathbb{V}$ be an n dimensional vector space over the field $\mathbb{F}$ with q elements. We have the following

1. If n = 1 then $dim_{\Gamma(\mathbb{V}_{\mathfrak{B}})}=q-1$.
2. If q=2 and $n\geq 2$, then $dim_{\Gamma(\mathbb{V}_{\mathfrak{B}})}=n$.
3. If q>2 and $n\geq 2$ then $dim_{\Gamma(\mathbb{V}_{\mathfrak{B}})}=\sum_{k=1}^{n}\binom{n}{k}((q-1)k-1)$.

Proof
1). By Theorem 1.1 $\Gamma(\mathbb{V}_{\mathfrak{B}})$ is complete graph of order q-1. Hence, $dim\Gamma(\mathbb{V}_{\mathfrak{B}})=q-1$.

2).For q=2 , a twin-set C_v is of length 1 for every element v *if* $\mathbb{V}$ Let $X=\{v_i:1\leq i\leq n\}$ where $S_{(v_i)}=\mathfrak{B}\setminus\{\alpha_i\}$. Now every two elements from $\Gamma(\mathbb{V}_{\mathfrak{B}})$ is resolved by some $v_k\in X$. By Lemma 2.2

$dim_{\Gamma(\mathbb{V}_{\mathfrak{B}})} \geq n$. Hence, we conclude that X is minimum resolving set of $\Gamma(\mathbb{V}_{\mathfrak{B}})$ and $dim_{\Gamma(\mathbb{V}_{\mathfrak{B}})} = n.$

3). For $q>2$, a twin-set C_v is of minimum length 2 for every element v of $\mathbb{V}$. By Lemma 1.2, a resolving set must contain all vertices, except one, in every twin set C_v. For fixed $k(1 \leq k \leq n)$ there are $\binom{n}{k}$ distinct twin sets and each twin sets have $(q-1)^k$ elements. Since q-1 choices and k places. Hence any resolving set in $\Gamma(\mathbb{V}_{\mathfrak{B}})$ must contain at least $\sum_{k=1}^{n}\binom{n}{k}((q-1)^k-1)$ elements. By Lemma 1.2 resolving set W contains $Y=\{v_i : S_{(vi)} = \mathfrak{B} \setminus \{\alpha_i\}\}$ for some $a_j \in \mathbb{F}$. Since, any two vertices with different skeleton is resolved by some $v_i \in Y$. Therefore $dim_{\Gamma(\mathbb{V}_{\mathfrak{B}})} = \sum_{k=1}^{n}\binom{n}{k}((q-1)k-1).$

Let n,q be integers. Let $\mathbb{V}$ be an n dimensional vector space over the field $\mathbb{F}$ with q elements. We have the following

1. If $n = 1$ then $ld_{\Gamma(\mathbb{V}_{\mathfrak{B}})} = q-1$
2. If $q = 2$ and $n \geq 2$, then $ld_{(\ )} \quad n$
3. If $q > 2$ and $n \geq 2$ $ld_{\Gamma(\mathbb{V}_{\mathfrak{B}})} = \sum_{k=1}^{n}\binom{n}{k}((q-1)k-1).$

Proof

1). By first part of Theorem 2.3 and Theorem 1.1 we get $ld_{\Gamma(\mathbb{V}_{\mathfrak{B}})} = q-1$.

2). Let X as defined in the second part of the proof in theorem 2.3 is minimum resolving set and also every element of $\Gamma(\mathbb{V}_{\mathfrak{B}})$ is adjacent to at least one element in c. Hence is also a dominating set so $ld_{\Gamma(\mathbb{V}_{\mathfrak{B}})} = n$.

(3). Let as defined in the third part of proof of the theorem 2.3 is dominating set. Since, every element of $\Gamma(\mathbb{V}_{\mathfrak{B}})$ is adjacent to at least one element in Y. Hence, $ld_{\Gamma(\mathbb{V}_{\mathfrak{B}})} = \sum_{k=1}^{n}\binom{n}{k}((q-1)k-1)$.

By definition of metric dimension, metric-locating-dominating set and locating-dominating set we observe that $\dim_G \leq mld_G \leq ld_G$. This relation proves the following corollary.

Corollary 2.5

Let n,q be integers. Let $\mathbb{V}$ be an n dimensional vector space over the field $\mathbb{F}$ with q elements. We have the following

(1) If n=1, then $mld_{\Gamma(\mathbb{V}_{\mathfrak{B}})} = q-1.$

(2) If $q - 2$ and, then $mld_{\Gamma(\mathbb{V}_{\mathfrak{B}})} = n.$

(3) If q > 2 and $n > 2$, then $mld_{\Gamma(\mathbb{V}_{\mathfrak{B}})} = \sum_{k=1}^{n}\binom{n}{k}((q-1)k-1).$

Theorem 2.6

Let n≥3, q ≥2 be integers. Let $\mathbb{V}$ be an n dimensional vector space over the field $\mathbb{F}$ with q elements then,

$pd_{\Gamma(\mathbb{V}_{\mathfrak{B}})} = n+(q+1)n.$

Proof

By Lemma 1.3 vertices form one twin-set is present in different partition of any resolving partition. The largest twin-set in $\mathbb{V}$ is skeleton of the vertices is $\mathfrak{B}$ and cardinality of the set is $(q+1)^n$. Thus $pd_{\Gamma(\mathbb{V}_{\mathfrak{B}})} \geq (q+1)n.$ Consider the partition $\Pi = \{\{v_1\},\{v_2\},\ldots,\{v_n\}, P_1, P_2, \ldots, P_{(q+1)n}\}$ where $v_i = \sum_{j\neq i}\alpha_j$ and each P_i contains exactly one vertex v from the twin-set of vertices having the skeleton $\mathfrak{B}$All other vertices of $\Gamma(\mathbb{V}_{\mathfrak{B}})$ is distributed using Lemma 1.3. Further, any two vertices from the same class of Π can be resolved by some v_i. Thus, Π is a resolving partition and $pd_{\Gamma(\mathbb{V}_{\mathfrak{B}})} \leq n+(q+1)n$. Next, we have to prove that $pd_{\Gamma(\mathbb{V}_{\mathfrak{B}})} \geq n+(q+1)n.$ Let $\Pi = \{A_1, A_2, \ldots, A_{n'} \, P_1, P_2, \ldots, P_{(q+1)}n\}$ be resolving partition. $pd_{\Gamma(\mathbb{V}_{\mathfrak{B}})} \geq (q+1)n$, one need to show that $n' \geq n$. Suppose any $v_i \in P_j$ with $1 \leq i \leq n$ and $1 \leq j \leq (q+1)^n$. Then there exists one element $u \in P_j$ with $S_u = \mathfrak{B}$. Now we have $r(u\,|\,\Pi) = r(v_i\,|\,\Pi)$ which is a contradiction to fact that Π is resolving partition. Suppose $V_i$and V_k are in same partition we get $r(v_i\,|\,\Pi) = r(v_k\,|\,\Pi)$ which is a contradiction. Hence each v_i in distinct partition other then P_j. Hence $n' \geq n$ this proves the theorem.

Decomposition of $\Gamma(\mathbb{V}_{\mathbf{B}})$

A decomposition of a graph G is a collection of edge-disjoint subgraphs $H_1, H_2, \ldots H_i$ of G belongs to exactly one H_i. For in this section we produce the results related to possible decomposition of $\Gamma(\mathbb{V}_{\mathfrak{B}})$.

Theorem 3.1

Let $n \geq 1$, $q \geq 1$ be integers. Let $\mathbb{V}$ be an n dimensional vector space over the field $\mathbb{F}$ with q elements then, $\Gamma(\mathbb{V}_{\mathfrak{B}})$ is decomposed into complete bipartite graphs.

Proof

Consider the partition of V is $V_i = \{v \in V : |S_v| = i\}$where $1 \leq i \leq n$. *Now consider the following three cases,*

Case 1

This case we characterize all the edges inside each V_i where $1 \leq i \leq n-1$

Subcase i

For $1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$ each element in V_i has no edge relation inside V_i.

Subcase ii

For $\left\lceil \frac{n}{2} \right\rceil \leq i \leq n-1$, now each $V_i$has $\binom{n}{i}$ twin sets with each twin set have q–1 elements. Elements of each twin set has adjacent to every element in $\binom{i}{n-i}$twin sets. Hence every twin set form the $\binom{i}{n-i}$ times $K_{(q-1)^i,(q-1)^i.}$ Therefore inside V_i is decomposed into

$\frac{\binom{n}{i}\binom{i}{n-i}}{2}$ times $K_{(q-1)^i,(q-1)^i.}$

Case 2

This case we characterize all the edges outside each V_i where $\left\lceil \frac{n}{2} \right\rceil \leq i \leq n-1.$Since there is edge relation between V_i for $1 \leq i \leq \frac{n}{2}.$

Sub Case i

Suppose n is odd, let $1 \le i \le \left\lceil \frac{n}{2} \right\rceil$ each vertex in V_i is center of star graph with $\sum_{k=0}^{n-i-1} (q-1)^i$ Pendent vertices. Hence, all the edges outside V_i is decomposed into $\bigcup_{i=\left\lceil \frac{n}{2} \right\rceil}^{n-1} (q-1)^i S \sum_{k=1}^{n-i-1} \binom{i}{k} (q-1)^i$

Sub Case ii

Suppose n is even, let $1 \le i \le \frac{n}{2} - 1$ each vertex in V_i is center of star graph with pendent $\sum_{k=0}^{n-i-1} (q-1)^i$ vertices. Hence, all the edges outside are decomposed into $\bigcup_{i=\left\lceil \frac{n}{2} \right\rceil}^{n-1} (q-1)^i S \sum_{k=1}^{n-i-1} \binom{i}{k} (q-1)^i$

Case 3

Let $i = n$ every element in V_i is adjacent to all other all elements in $\mathbb{V}$. Since, all the above cases all the edges are decomposed into union of $(q-1)^n$ star graph with each of order (q^n-1).

Hence, $\Gamma(\mathbb{V}_{\mathfrak{B}})$ is decomposed into complete bipartite graphs.

Results on $\Gamma(\mathbb{V})$

In this section, we found the structure of $\Gamma(\mathbb{V})$ and genus of linear dependent graph of vector space. Also, we found the power domination number of $\Gamma(\mathbb{V})$.

Theorem 4.1: $\Gamma(\mathbb{V})$ is isomorphic to $K_{q-1}^{q^{n-1}+q^{n-2}+\ldots+1}$

Proof

We observe that 0 is adjacent to all the vertices of $\Gamma(\mathbb{V})$. Each one-dimensional subspace of $\mathbb{V}$ form a complete subgraph of $\Gamma(\mathbb{V})$. Also is in every subspace of $\mathbb{V}$. Since total number of one-dimensional subspaces of $\mathbb{V}$ is $q^{n-1}+q^{n-2}+\ldots+1$ .Therefore $\Gamma(\mathbb{V})$ is one point union of $q^{n-1}+q^{n-2}+\ldots+1$ copies complete graph of order q-1. Hence $\Gamma(\mathbb{V})$ is isomorphic to $K_{q-1}^{q^{n-1}+q^{n-2}+\ldots+1}$.

Note that the eccentricity of $0 \in \Gamma(\mathbb{V})$ is 1 and the eccentricity of nonzero element of $\Gamma(\mathbb{V})$ is 2. By Theorem 4.1 we have the following theorems.

Corollary 4.2: $\Gamma(\mathbb{V})$ is decomposed into star and complete graph.

By the observation 1 in [2] is any graph $G, 1 \le \gamma_p(G) \le \gamma(G)$ and by Theorem 1.4, we have the following theorem.

Theorem 4.3: Power domination number $\gamma_p(\Gamma(\mathbb{V}))$ is 1.

Theorem 4.4: Radius of the graph $\Gamma(\mathbb{V})$ is 1.

Theorem 4.5: $g(\Gamma(\mathbb{V})) = (q^{n-1}+q^{n-2}+\ldots+1)\left\lceil \frac{(q-3)(q-4)}{12} \right\rceil$

Proof

By Theorem 4.1 $\Gamma(\mathbb{V})$ is $K_{q-1}^{q^{n-1}+q^{n-2}+\ldots+1}$. Let $G_1 = K_q$ (elements of $\mathbb{V}$ generated by single element some $v \in \mathbb{V}$) and $G_2 = K_{q-1}^{q^{n-1}+q^{n-2}+\ldots+2}$. Hence we get $\Gamma(\mathbb{V}) = G_1 \cup G_2$ and $G_1 \cap G_2 = \varnothing$ then $g(\Gamma(\mathbb{V})) \leqq g(G_1) + g(G_2)$. By theorem 1.5 $g(G_1) = \left\lceil \frac{(q-3)(q-4)}{12} \right\rceil$ and $g(\Gamma(\mathbb{V})) \leqq \left\lceil \frac{(q-3)(q-4)}{12} \right\rceil + g(G_2)$. Now to find $g(G_2)$ Let $H_1 = Kq$ and $H_2 = K_{q-1}^{(q^{n-1}+q^{n-2}+\ldots+1)-2}$ Hence we get $G_2 = H_1 \cup H_2$ and $H_1 \cap H_2 = \varnothing$ then $g(G_2) \leqq g(H_1) + g(H_2)$. By Theorem 1.3, $g(H_1) = \left\lceil \frac{(q-3)(q-4)}{12} \right\rceil$ and $g(\Gamma(\mathbb{V})) \leqq 2\left\lceil \frac{(q-3)(q-4)}{12} \right\rceil + g(H_2)$. Now find $g(H2)$ continuing this process up to $q^{n-1}+q^{n-2}+\ldots+1$ we get $g(\Gamma(\mathbb{V})) \leqq (q^{n-1}+q^{n-2}+\ldots+1)\left\lceil \frac{(q-3)(q-4)}{12} \right\rceil$. Similarly, we can prove $g(\Gamma(\mathbb{V})) \geqq (q^{n-1}+q^{n-2}+\ldots+1)\left\lceil \frac{(q-3)(q-4)}{12} \right\rceil$ by using lemma 1.7. $g(\Gamma(\mathbb{V})) = (q^{n-1}+q^{n-2}+\ldots+1)\left\lceil \frac{(q-3)(q-4)}{12} \right\rceil$.

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