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Resolution of the Fundamental Equation of the Mathematical Formulation of Nuclear Collision Mechanisms Model

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ABSTRACT

The fundamental equation of the model titled "mathematical formulation of nuclear collision mechanisms" [1] was solved. The solution is compounded of two steps, the first step consists in calculating the values of the parameters of the mechanism-operator underlying a given nuclear collision and the second step, consists in deducing the open channels for this collision. Knowing that the equation is not classic, its resolution has required the development of an innovative mathematical method that sets on the use of physical constraints of the problem.

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Introduction

A reflexion on the mathematical theory subtending the model [1] has achieved there are some concepts worthing developed. Among these latest, there is a verry important one who is the evolution equation of the studied system. Let's reminding that this fundamental equation of the model was already used in the paper [1] but in a special circumstance in which both the in and out channels were given and the goal was to determine the mechanisms occurring along the collision. In this work, we try to lead with this equation by rising the following problem: giving a system in collision (in other words, given the entrance channel), which mechanisms can take place during the collision, and which exit channels can be opened? Responding these questions means solving the fundamental equation. For this purpose, the strategy adopted is focalizing on the technical side of the resolution of the equation in the paragraph titled "Resolution of the fundamental equation", this latest is preceded by another one where some algebraic properties of the sets $\boldsymbol{E}_2$ (system-states set), $\boldsymbol{\vartheta}$ (set of mechanisms) and $\boldsymbol{C}$ (constraint's ensemble of the problem) are presented. Knowing that the equation is not a classical one, the view adopted for its resolution rests on consideration of physical constraints as a principal tool to solve it.

Generalities

The fundamental equation of the model [1] is: $\theta E = S$, it is an algebraic equation. Its components are (θ, E, S), where θ is an operator, E and S are particular states of the physical system considered. In following, allow me providing some clarifications on these components.

System-States Set

Among the components of the equation the ensemble of the states of the system noted E_2 in [1]. Allow me remind also that this equation describes the evolution of a physical system defined as any couple of nuclei who's the composition is specified (two isotopes) and are in collision. This system passes by some states along its evolution. Each system-state is described mathematically by a board composed of two columns, each column represents a nucleus throw its neutron and proton numbers. A collision begins by an initial state $E = \begin{pmatrix} n_{e1} & n_{e2} \\ z_{e1} & z_{e2} \end{pmatrix}$ of the system (entrance channel) and finishes by a final state $S = \begin{pmatrix} n_{s1} & n_{s2} \\ z_{s1} & z_{s2} \end{pmatrix}$ (exit channel) and during the collision the system passes by a series of intermediate states $\left\{ \varphi = \begin{pmatrix} n' & n'' \\ z' & z'' \end{pmatrix} \right\}$. Hence, E and S are particular elements of $\boldsymbol{E}_2$.

Mechanisms Set

θ is the mathematical formulation of the physical mechanism [1] taking place when a couple of nuclei collide, evolute to finish by giving another couple of nuclei.

Mathematically, θ is an element of a set ϑ whom is the ensemble of the mechanisms relative to a given system in collision. $(\boldsymbol{\vartheta}, \boldsymbol{o})$, with $\boldsymbol{o}$ the usual product of operators has an abelian group structure and the application of ϑ on $\boldsymbol{E}_2$ is a group action [i,ii]. Moreover, by construction [1] ϑ is a finitely generated abelian group and its generators family is $\left\{ G_0^1, B_0^1, D_0^1, H_0^1 \right\}$

Constraints

Because of their role in the resolution of the fundamental equation on one hand, and saw the large number of constraints we can consider in the domain of nuclear reactions on other hand. It is necessary to evoke which constraints will be considered along the resolution of the fundamental equation and how they could be used.

The constraints used in following, are the relations imposed to certain system-states, principally the entrance and the exit channels, by physical considerations. In this work, the conservation laws (as conservation of charge and neutrons numbers laws, sign of the elements of the states and others constraints that will be precited when needed) that are used are seen as a particular case of a largest concept that is constraint. This latest is not necessarily confirmed experimentally as it is the case of the conservation laws but it is necessary for the resolution of the equation.

Resolution of the Fundamental Equation

After these precisions about the sets: $\boldsymbol{\vartheta}$, $\boldsymbol{E}_2$, **C** let's begin the resolution.

The fundamental equation is:

$$\theta E = S \qquad (1)$$

Explicitly, (1) is written as

$$G_g^{g'} B_b^{b'} D_d^{d'} H_h^{h'} \begin{pmatrix} n_{e1} & n_{e2} \\ z_{e1} & z_{e2} \end{pmatrix} = \begin{pmatrix} n_{s1} & n_{s2} \\ z_{s1} & z_{s2} \end{pmatrix} \qquad (2)$$

Where $E = \begin{pmatrix} n_{e1} & n_{e2} \\ z_{e1} & z_{e2} \end{pmatrix}$ and $S = \begin{pmatrix} n_{s1} & n_{s2} \\ z_{s1} & z_{s2} \end{pmatrix}$

Before solving the equation, let's making the notations used above lighten, so E and S will be noticed: $E = \begin{pmatrix} n_{01} & n_{02} \\ z_{01} & z_{02} \end{pmatrix}$ $S = \begin{pmatrix} n_1 & n_2 \\ z_1 & z_2 \end{pmatrix}$.

Hence, the equation (2) becomes:

$$G_g^{g'} B_b^{b'} D_d^{d'} H_h^{h'} \begin{pmatrix} n_{01} & n_{02} \\ z_{01} & z_{02} \end{pmatrix} = \begin{pmatrix} n_1 & n_2 \\ z_1 & z_2 \end{pmatrix} \qquad (3)$$

Then, let's write the equation (3) as a system of four equations according the action of θ on E in the original paper (the equations (4) of the paragraph ''C Mechanisms calculus'') [1]:

$$\begin{cases} n_1 = n_{01} - g + h' \\ z_1 = z_{01} + g' - b \\ n_2 = n_{02} + d' - h \\ z_2 = z_{02} + b' - d \end{cases}$$

Or equivalently

$$\begin{cases} h' = n_1 - n_{01} + g \\ g' = z_1 - z_{01} + b \\ d' = n_2 - n_{02} + h \\ b' = z_2 - z_{02} + d \end{cases} \qquad (4)$$

By replacing theses quantities in θ and simplifying, it comes

$$\theta = G_0^{z_1 - z_{01}} B_0^{z_2 - z_{02}} D_0^{n_2 - n_{02}} H_0^{n_1 - n_{01}}$$

Let's note these quantities as following:

$$\begin{cases} z_1 - z_{01} = \alpha \\ z_2 - z_{02} = \beta \\ n_2 - n_{02} = \gamma \\ n_1 - n_{01} = \mu \end{cases} \qquad (5)$$

A reading of equations (4) shows, there are two sets of inconnus that have not the same nature. A first set $(\alpha, \beta, \gamma, \mu)$, that determines the parameters of the mechanism-operator. And a second set (n_1, z_1, n_2, z_2), that determines those of the final state of the colliding system. According to the equations (4) the determination of one of the two sets of inconnus will permit to deduce the other set. Given an exit channel or equivalently giving the values of the set (n_1, z_1, n_2, z_2) that situation was treated in the paper [1]. Currently, we face the opposite situation in which no values of the inconnus are given. To solve the equations-system (4) a method was found, this one rests on conservation laws. This idea comes from the following remark. There is a relation between the constraints imposed to the elements of $\boldsymbol{E}_2$ and the mechanisms (elements of ϑ). In this way, when a constraint $\boldsymbol{c}_i$ is taken in account (is consequently considered a subset $\boldsymbol{E}_2^{c_i}$ of $\boldsymbol{E}_2$ containing only the elements responding to criteria imposed by the constraint), both the mechanisms and the system-states sets are restricted respectively to subsets $\boldsymbol{\vartheta}_{c_i}$ and $\boldsymbol{E}_2^{c_i}$ of $\boldsymbol{\vartheta}$ and $\boldsymbol{E}_2$. The consideration of another constraint cj will in turn restrict ϑ and $\boldsymbol{E}_2$ to the subsets $\boldsymbol{\vartheta}_{c_j}$ and $\boldsymbol{E}_2^{c_j}$.

The consideration of two constraints $\boldsymbol{c}_i$ and $\boldsymbol{c}_j$ simultaneously, will restrict the study to the intersection $\boldsymbol{E}_2^{c_i} \cap \boldsymbol{E}_2^{c_j}$ and $\boldsymbol{\vartheta}_{c_i} \cap \boldsymbol{\vartheta}_{c_j}$ of the subsets relative to each of the constraints respectively in $\boldsymbol{E}_2$ and $\boldsymbol{\vartheta}$.

The following example illustrates the proposals above, let's consider the equations (4) above. If none constraint was taken in account, the quantities $(\alpha, \beta, \gamma, \mu)$ can take any entire relative values. Then, the components of the exit channel will have no physical meaning because they may be negative.

If we want to have only the mechanisms obeying the constraint $\boldsymbol{c}_2 : (z_{01} + z_{02} = z_1 + z_2)$ (conservation law of charge), we consider equivalently the subset $\boldsymbol{E}_2^{c_2}$ of $\boldsymbol{E}_2$.

We act $\theta = G_0^{\alpha} B_0^{\beta} D_0^{\gamma} H_0^{\mu}$ on an entry $E = \begin{pmatrix} n_{01} & n_{02} \\ z_{01} & z_{02} \end{pmatrix}$ according the fundamental equation (1)

$$\theta \begin{pmatrix} n_{01} & n_{02} \\ z_{01} & z_{02} \end{pmatrix} = \begin{pmatrix} n_{01} + \mu & n_{02} + \gamma \\ z_{01} + \alpha & z_{02} + \beta \end{pmatrix} = \begin{pmatrix} n_1 & n_2 \\ z_1 & z_2 \end{pmatrix}$$

And by equalising the elements of the last equality we obtain

$$\begin{cases} n_{01} + \mu = n_1 \\ z_{01} + \alpha = z_1 \\ n_{02} + \gamma = n_2 \\ z_{02} + \beta = z_2 \end{cases}$$

Then, by adding the second and the fourth equations and taking in account $\boldsymbol{c}_2$, we have $\alpha = -\beta$. Thus, $\boldsymbol{\vartheta}$ is reduced to $\boldsymbol{\vartheta}_{c_2}$ whose elements are $G_0^{-\beta} B_0^{\beta} D_0^{\gamma} H_0^{\mu}$ that forms a subgroup of the group ϑ [iii].

The consideration of another constraint for example $\boldsymbol{c}_1$: $(n_{01} + n_{02} = n_1 + n_2)$ will reduce $\boldsymbol{E}_2$ to $\boldsymbol{E}_2^{c_1}$ and ϑ to $\boldsymbol{\vartheta}_{c_1} = \left\{ G_0^{\alpha} B_0^{\beta} D_0^{\gamma} H_0^{-\gamma} \right\}$.

The consideration of c_1 and c_2 simultaneously, will reduce E_2 to the intersection $E_2^{c_1} \cap E_2^{c_2}$ and imposes the subgroup ($\vartheta_{c_1} \cap \vartheta_{c_1}$) of ϑ whose elements are: $G_0^{-\beta} B_0^{\beta} D_0^{\gamma} H_0^{-\gamma}$.

Use of the Usual Laws

Considering the remark of the paragraph above, the first physical well-known constraints that we have taken in account are conservation laws of neutrons and protons numbers, named c_1 and c_2 in example above. The consequence of the first constraint above on the mechanisms is($\mu = -\gamma$) [iv] and the effect of the second conservation law is ($\alpha = -\beta$) [v]. Hence, the set of mechanisms is reduced to $G_0^{-\beta} B_0^{\beta} D_0^{\gamma} H_0^{-\gamma}$.

If we replace α and μ by their values in the equations (4) we have

$$\begin{cases} z_1 - z_{01} = -\beta \\ z_2 - z_{02} = \beta \\ n_2 - n_{02} = \gamma \\ n_1 - n_{01} = -\gamma \end{cases} \qquad (6)$$

Research of other Constraints

The equations (6) show that the number of inconnus has decreased by two unites and became two instead four. Inspired by the efficiency of this method, the consideration of more constraints will permit to establish a link between γ and β. To find these constraints, let's introduce some results concerning them.

As mentioned above, each constraint whom an element of $\mathcal{P}(C)$ defines a subgroup of ϑ and inversely, each subgroup of ϑ defines a subset of C (classes of constraints set) [vi].

The consideration of the following constraint:

$$n_1 - z_2 = n_{01} - z_{02} \qquad (7)$$

gives $\gamma = -\beta$.

And, the subgroup of ϑ obeying to these tree constraints is $\theta = G_0^{-\beta} B_0^{\beta} D_0^{-\beta} H_0^{\beta}$. So, the equations (6) become

$$\begin{cases} z_1 - z_{01} = -\beta \\ z_2 - z_{02} = \beta \\ n_2 - n_{02} = -\beta \\ n_1 - n_{01} = \beta \end{cases} \qquad (8)$$

Another set of physical constraints are $n_i \geq 0$ and $z_i \geq 0$ where i= 1,2

So, according to equations (8) $-z_{02} \leq \beta \leq z_{01}$ and

$-n_{01} \leq \beta \leq n_{02}$ then, the interval to be considered is the intersection of theme, and because of the nuclei in entrance channel are in general stable $n_{0i} \geq z_{0i}$ where i =1,2 the intersection of the two intervals is the interval $[-z_{02}, z_{01}]$.

Open Channels

The determination of the open channels or the second set of inconnus (n_1,z_1, n_1,z_2) mentioned in the first paragraph comes by the application of the mechanisms found above to the entrance channel.

In fact, let's acting $\theta = G_0^{-\beta} B_0^{\beta} D_0^{-\beta} H_0^{\beta}$ on $\begin{pmatrix} n_{01} & n_{02} \\ z_{01} & z_{02} \end{pmatrix}$

It come

$\theta \begin{pmatrix} n_{01} & n_{02} \\ z_{01} & z_{02} \end{pmatrix} = \begin{pmatrix} n_{01} + \beta & n_{02} - \beta \\ z_{01} - \beta & z_{02} + \beta \end{pmatrix}$ where β takes the values of the interval $[-z_{02}, z_{01}]$ say $(z_{02} + z_{01} + 1)$ values that is also the number of open channels for the system ${}^{n_{01}+z_{01}}_{z_{01}}X + {}^{n_{02}+z_{02}}_{z_{02}}Y$ considered in collision.

Results and Discussion

Technically, the resolution consists in finding the possible mechanisms for a given entrance channel and the deduction of the open channels for each mechanism. However, there are four ideas underlying this method that must be reminded.

✓ The key of the resolution of the fundamental equation is the remark made at the biggening of the second paragraph. This remark shows the relation existing between the constraints and the mechanisms. I wish like to remind it because it is a no classical method. In fact, classically the constraints of the problem are used to determine constants imposed physically (as boundary conditions) not for solving the equations of the problem. Inversely, in this problem, where there are no constants to be determined. the constraints permit the determination of the values of the evolution-operator whom initially inconnus.

✓ It is also another idea that must be evoked also. The logic of the resolution let seeing that the solutions found (mechanisms and exit channels) are the consequences of the constraints fixed by the physicist. Hence, the constraints used above, in the resolution of the equations, is only a choice among others. The possible confusions can be clarified by the following assumption.

The relation existing between the parts-set $\mathcal{P}(C)$ of the constraint's ensemble and the subgroups-ensemble of the group (ϑ,o) can be seen mathematically as an application between them. If we note this application g,

$$g : \mathcal{P}(C) \rightarrow \{\vartheta_c\}$$

$$c \rightarrow g(c) = \vartheta_c$$

Because g is bijective [vii] we can say, for each constraint c (element of $\mathcal{P}(C)$) correspond a subgroup of ϑ and reversely, for each subgroup ϑ_c of ϑ correspond a constraint c.

✓ Mathematically, the resolution of the evolution equation amounts to the reduction of the mechanism-group ϑ considered initially to its minimal subgroup.

Dedicates

I am honoured to dedicate this work to my son Mohammed al Mahdi and my daughter Hanaa, this girl that always cherished me and makes my jealous of my fellows by asking me each time When will we hear of your work?

i. Giving two mechanisms θ_1 and θ_2, knowing that $\theta_1 = G_0^{\alpha_1} B_0^{\beta_1} D_0^{\gamma_1} H_0^{\mu_1}$ and $\theta_2 = G_0^{\alpha_2} B_0^{\beta_2} D_0^{\gamma_2} H_0^{\mu_2}$, naturally the composition of these mechanisms is:

$\theta_1 \theta_2 = G_0^{\alpha_1+\alpha_2} B_0^{\beta_1+\beta_2} D_0^{\gamma_1+\gamma_2} H_0^{\mu_1+\mu_2} =$

$$G_0^{\alpha_2+\alpha_1}B_0^{\beta_2+\beta_1}D_0^{\gamma_2+\gamma_1}H_0^{\mu_2+\mu_1}=\theta_2\theta_1$$

because the powers are relative numbers. So, this composition law is interne and commutative.

It is obvious that the law has a neutral element, $\theta_0=G_0^0B_0^0D_0^0H_0^0$

and finally, each element $\theta=G_0^\alpha B_0^\beta D_0^\gamma H_0^\mu$ has an opposite $\theta'=G_0^{-\alpha}B_0^{-\beta}D_0^{-\gamma}H_0^{-\mu}$.

ii. The operation of the group ϑ on E_2 is a group action. In fact, the neutral element of ϑ is

$\theta_0=G_0^0B_0^0D_0^0H_0^0$ and $\theta_0=G_0^0B_0^0D_0^0H_0^0$ applied to a given board (element of the system states set) $\varphi=\begin{pmatrix}n_1 & n_2\\ z_1 & z_2\end{pmatrix}$ gives φ, and the image by $\theta_1=G_0^{\alpha_1}B_0^{\beta_1}D_0^{\gamma_1}H_0^{\mu_1}$ of

$$\theta_2=G_0^{\alpha_2}B_0^{\beta_2}D_0^{\gamma_2}H_0^{\mu_2}\begin{pmatrix}n_1 & n_2\\ z_1 & z_2\end{pmatrix}=\begin{pmatrix}n_1+\mu_2 & n_2+\gamma_2\\ z_1+\alpha_2 & z_2+\beta_2\end{pmatrix}$$ gives

$$G_0^{\alpha_1}B_0^{\beta_1}D_0^{\gamma_1}H_0^{\mu_1}\begin{pmatrix}n_1+\mu_2 & n_2+\gamma_2\\ z_1+\alpha_2 & z_2+\beta_2\end{pmatrix}=\begin{pmatrix}n_1+\mu_2+\mu_1 & n_2+\gamma_2+\gamma_1\\ z_1+\alpha_2+\alpha_1 & z_2+\beta_2+\beta_1\end{pmatrix}=$$

$$G_0^{\alpha_1+\alpha_2}B_0^{\beta_1+\beta_2}D_0^{\gamma_1+\gamma_2}H_0^{\mu_1+\mu_2}=(\theta_1\theta_2)\varphi$$

iii. Let me name this set ϑ_c, $\theta_0=G_0^0B_0^0D_0^0H_0^0\in\vartheta_c$.

Giving two elements $\theta_1=G_0^{\alpha_1}B_0^{\beta_1}D_0^{-\alpha_1}H_0^{\mu_1}$ and

$\theta_2=G_0^{\alpha_2}B_0^{\beta_2}D_0^{-\alpha_2}H_0^{\mu_2}$ $\theta_1\theta_2^{-1}$ is also an element of ϑ_c.

iv. considering the fundamental equation [1]. Writing it explicitly, it takes the form:

$$\theta\begin{pmatrix}n_{01} & n_{02}\\ z_{01} & z_{02}\end{pmatrix}=\begin{pmatrix}n_{01}+\mu & n_{02}+\gamma\\ z_{01}+\alpha & z_{02}+\beta\end{pmatrix}=\begin{pmatrix}n_1 & n_2\\ z_1 & z_2\end{pmatrix}$$

Equalizing the members of the last equality, it comes:

$$\begin{cases}z_1-z_{01}=\alpha\\ z_2-z_{02}=\beta\\ n_2-n_{02}=\gamma\\ n_1-n_{01}=\mu\end{cases}$$

adding the third and the fourth equations and by taking in account the conservation of neutrons we deduce: $\mu=-\gamma$

v. This time, we add the first equation to the second equation and then we deduce: $\alpha=-\beta$

vi. Because of the finite cardinal of subgroups set of ϑ that are:

$$\left[\{\varnothing\},\{G^\alpha\},\{B^\beta\},\{D^\gamma\},\{H^\mu\},\{G^\alpha B^\beta\},\{G^\alpha D^\gamma\}\{G^\alpha H^\mu\},\{B^\beta D^\gamma\},\{B^\beta H^\beta\},\{D^\gamma H^\mu\},\{G^\alpha B^\beta D^\gamma\},\{G^\alpha B^\beta H^\mu\},\right]\{B^\beta D^\gamma H^\mu\},\{G^\alpha D^\gamma H^\mu\},\{G_0^\alpha B_0^\beta D_0^\gamma H_0^\mu\}$$

the demonstration was done for each case separately. For example, considering the subgroup $\{B^\beta D^\gamma\}$ means that α = μ = 0 and by taking in account the equations (4),

we have $\begin{cases}z_1=z_{01}\\ z_2-z_{02}=\beta\\ n_2-n_{02}=\gamma\\ n_1=n_{01}\end{cases}$ so the first and forth equations give conservation laws.

Inversely, if we considered as conservation laws: $z_1=z_{01}$ and $n_1=n_{01}$ it is obvious that we have the subgroup $\{B^\beta D^\gamma\}$.

The same reasoning can be informed for all subgroups cited above.

vii. $\boldsymbol{g}$ Is surjective because for any subgroup of $\boldsymbol{\vartheta}$ correspond a constraint and for the subgroup ϑ himself correspond $\{\varnothing_c\}$.

Is injective because for any couple of subgroups θ_i and θ_j of ϑ, $g(c_i)=g(c_j)\ \langle=\rangle\ \theta_i=\theta_j$.

In fact, for an entry E, $\theta_iE=\theta_jE\ \langle=\rangle$

$$\begin{pmatrix}n_{01}+\mu_i & n_{02}+\gamma_i\\ z_{01}+\alpha_i & z_{02}+\beta_i\end{pmatrix}=\begin{pmatrix}n_{01}+\mu_j & n_{02}+\gamma_j\\ z_{01}+\alpha_j & z_{02}+\beta_j\end{pmatrix}$$

this means, the constraint calculated by the mechanism θ_i or θ_j is the same constraint what ends the proof.

References

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