

## Theoretical Foundations of the Physical-Mathematical Model of the Non-Stationary Combustion Process of Solid Fuel and Derivation of Methods for the Calculation of Non-Stationary Velocity

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### ABSTRACT

This study investigates the physico-chemical processes in the gas phase, considering the methods for calculating of burning rate of solid propellant and physico-mathematical model of non-stationary combustion of solid fuel, we study the equation for heat propagation in the k-phase, the specification of the initial and boundary condition, the fundamental solution of the heat conduction equation in the k-phase is obtained. We consider the equation for calculating non-stationary combustion velocity, the stationary combustion velocity in the presence of catalysts in the fuel composition, the method for calculating constants. We investigate the analytical solution of the linearized integral equation for the burning rate, we determine the particular solutions of the linearized integral equation and we found the numerical and approximate solutions of the nonlinear integral equation for the burning velocity.

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**Received:** August 10, 2024; **Accepted:** September 11, 2024; **Published:** December 12, 2024

**Keywords:** Solid Fuel, Combustion Velocity, Physical-Mathematical Model, Ballistic Missiles, Intra-Ballistic Characteristic, Mathematical Methods, Numerical Methods,  $k$ -Phase

### The List of Abbreviations, Symbols, and Notations

#### Abbreviations

**SRE:** Solid Rocket Engine

**IBC:** Intra-Ballistic Characteristics

**k Phase:** Condensed Phase (Solid Fuel)

#### Key Notations

$p$  – Pressure

$u$  – Burning Rate

$t$  – Time

$x$  – Distance in the Coordinate System Related to the Combustion Surface

$T$  – Temperature

$f$  – Temperature Gradient

$\alpha$  – Thermal Conductivity Coefficient

$\lambda$  – Thermal Conductivity Coefficient

$c$  – Heat Capacity

$\rho$  – Density

$E$  – Activation Energy

$H$  – Pre-Exponential Factor

$R$  – Universal Gas Constant

$Q$  – Heat of Reaction

$A$  – Concentration

$Z$  – Molar Fraction

$\varepsilon$  – Heat Quantity

### Symbols

$o$  – Steady-State Conditions

$s$  – Combustion Surface

$s_o$  – Steady-State Conditions at the Combustion Surface

$g$  – Gas Phase

$N$  – Initial Conditions

### Introduction

Modeling solid fuel combustion processes is a complex task that involves the interaction of various physical and chemical phenomena. Researchers use mathematical models to understand and predict the behavior of solid fuel combustion in different systems, such as industrial boilers, power plants, and furnaces. Here are some key aspects of the modeling process:

- **Mass and Energy Balances:** Mathematical models often begin with mass and energy balances to describe the overall combustion process. These balances account for the fuel, oxidizer, and products of combustion, as well as heat transfer mechanisms;
- **Chemical Reactions:** solid fuel combustion involves complex chemical reactions. Models include reaction kinetics to describe how fuel particles decompose, release volatile gases, and react with oxygen;
- Detailed reaction mechanisms for different types of solid fuels are essential for accurate modeling;
- **Heat and Mass Transfer:** heat and mass transfer between the solid fuel, gas phase, and surrounding surfaces play a crucial role. Models consider convective and radiative heat transfer, as well as mass transfer of gases within the combustion chamber;

- **Particle Dynamics:** in solid fuel combustion, the behavior of individual fuel particles is important. Models may account for particle size distribution, shape, and trajectory;
- **Particle Dynamics Influence Combustion Efficiency and Pollutant Formation:** pollutant formation modeling includes the prediction of pollutant formation, such as NO<sub>x</sub>, SO<sub>x</sub>, and particulate matter. Understanding the factors influencing pollutant formation helps in designing cleaner and more efficient combustion processes;
- **Computational Fluid Dynamics (CFD):** CFD techniques are commonly used for simulating the fluid flow, temperature distribution, and species concentrations within combustion chambers. Advanced CFD models provide insights into the three-dimensional and transient nature of combustion processes;
- **Validation and Experimental Data:** models need to be validated against experimental data to ensure their accuracy and reliability. Continuous improvement and refinement of models are ongoing processes based on new experimental findings. The field of solid fuel combustion modeling is dynamic, with ongoing research focused on improving the accuracy and applicability of models to various practical scenarios. Researchers are also exploring sustainable and cleaner combustion technologies, which may involve alternative solid fuels and novel combustion processes.

Success in space research and practical applications hinges largely on advancements in rocket engine development, with a particular emphasis on the successful execution and theoretical understanding of solid fuel combustion processes. This involves refining methods for calculating engine characteristics. Currently, a substantial body of scientific work is dedicated to theoretically describing non-stationary processes occurring during the combustion of solid fuel in rocket engine chambers.

The most extensively studied aspect of non-stationary solid fuel combustion theory involves the investigation of vibrational processes and the rigorous analysis of combustion harshness. Esteemed researchers such as Ya.B. Zeldovich, B.V. Novozhilov, V.F. Prisnyakov and others, both from Russia and abroad, have delved into this critical area. However, the existing theoretical framework falls short in addressing the imperative for the development of solid fuel rocket engines, primarily due to the lack of practical methods for calculating non-stationary combustion rates.

This limitation arises from the inherent complexity of combustion processes, especially when catalysts are introduced. Catalysts significantly alter the relationship between solid fuel combustion rates and pressure, adding an extra layer of intricacy. Furthermore, the underutilization of findings from related scientific fields compounds this challenge. For instance, the theory of chain-branching reactions, which has proven successful in liquid fuel combustion theory and was developed by Academician Semenov, has not been effectively incorporated into solid fuel combustion theory.

The combustion velocity of solid fuel within the Intra Ballistic characteristic (IBC) of a solid fuel rocket motor (RMSF) is determined by the law of burning rate, which establishes its dependence on pressure and initial temperature. This law is derived through experimental burning of fuel samples under stationary conditions. Stationary burning rate laws are deemed appropriate when the crater time of the process is sufficiently large, approximately around  $10^{-1}$  s. However, under non-stationary

conditions, such as rapid pressure changes in RMSF occurring within a crater time of approximately  $10^{-3}$  s or less, the burning rate varies. In such instances, the burning rate cannot instantly conform to the value calculated by stationary laws. It requires some time during which complex processes occur, leading to a restructuring of the fuel burning front. These processes occur during the transition to the operating mode, pressure drop at the end of operation, and throughout the entire operational cycle of certain types of RMSF.

It is essential to consider the non-stationary nature of the burning rate of solid fuel when calculating intra-ballistic characteristics. This necessity arises from the prevalent use of rocket engines operating in “impulse” mode, where the engine functions for a duration of 5-50 milliseconds. Such engines find application in guidance systems for aircraft, ejector seats, and launch engines for diverse purposes.

The theory of Solid Fuel Combustion (SFC) is a standalone and well-developed branch of science. However, the internal ballistic calculations of specific Solid Rocket Motors (SRM), including those with “impulse” characteristics, are currently based solely on dependencies of burning rate on pressure obtained experimentally under stationary conditions.

The current situation arises from a twofold challenge. Firstly, there is uncertainty among developers of solid propellant rocket missiles regarding the necessity of incorporating the non-stationary component of burning rate. Secondly, there is a lack of specific methods designed to calculate non-stationary burning rates, especially tailored for their integration into the system of equations governing the internal ballistics of solid rocket motors.

The objective of this work is to bridge the existing gap between academic theories of combustion and the practical development of specific Rocket Motor Solid Propellants (RMSP). The ultimate goal is to devise a calculation method for the internal ballistics of solid rocket motors that explicitly takes into account the non-stationarity of the burning rate.

Analyzing the outcomes of computations employing this approach in contrast to experimental data will provide insights into the criteria that, once satisfied, permit the exclusion of non-stationarity considerations in calculating internal ballistics. The lack of established calculation dependencies has hitherto hindered the formulation of such criteria.

By the present time, a significant number of scientific works by researchers like Ya.B. Zeldovich, B.V. Novozhilov, V.F. Prisnyakov, both from Russia and abroad, have been published, focusing on the theoretical description of the processes occurring during the combustion of solid propellants in the combustion chambers of rocket engines. However, the results of the existing theory of non-stationary combustion have not found widespread application in the development of Solid Rocket Motors (SRMs).

For the calculation of the internal ballistics characteristics (IBC) of SRMs, burning rate laws are used. These laws describe the burning rate of solid propellants as functions of pressure and initial temperature, which have been experimentally obtained under stationary conditions. This approach is entirely justified when the characteristic time of pressure change  $(\frac{p}{dp/dt})$  is sufficiently large, on the order of  $10^{-1}$  s or more.

However, the real combustion rate may differ from the value calculated using stationary laws if the pressure change process occurs rapidly, i.e., within a characteristic time on the order of  $10^{-3}$  s or less. Such a process occurs during startup, pressure drop at the end of operation, and throughout the operational cycle of some types of SRMs, especially in the case of impulse SRMs with a firing duration of 5-50 milliseconds used in crew escape systems (CES) for aircraft and spacecraft. When CES is engaged, an error of more than 10 percent in the calculations of acceleration or its rate of increase is unacceptable, and, consequently, an error of more than 5 percent in calculating the burning rate of SRPs is also unacceptable.

Calculations for the IBC of such engines, especially during startup, must take into account all possible factors that can influence the internal chamber characteristics and, therefore, the magnitude of acceleration and its rate of increase, to which the human body is particularly sensitive. In this case, for a more accurate IBC calculation, the stationary burning rate law must be replaced with a relationship that links the non-stationary burning rate and non-stationary pressure.

In the existing theory of non-stationary SRP combustion, there was no question of developing specific methods for calculating the dependence of the non-stationary burning rate on pressure. When theoretically studying the combustion processes of solid propellants, either a time interval is considered in which the influence of initial conditions on the course of the process is negligible, or the problem statement is so general that obtaining a specific solution is not feasible.

In the existing theory of non-stationary combustion of solid fuel, the question of developing specific methods for expenditure dependency of non-stationary burning rate on pressure throughout the entire process was not addressed. The theoretical investigation considered the non-stationarity over a time interval where the influence of initial conditions on the process was negligible. Consequently, the curve problem with initial conditions for the heat conduction equation describing heat propagation in the  $k$ -phase was not posed in the study of this issue. Thus, creating a theoretical foundation for studying the non-stationary burning rate of solid fuel, grounded on accumulated results in this field and suitable for a more accurate calculation of the internal ballistics characteristics of solid rocket propellants, appears highly relevant.

Thus, the creation of a theoretical foundation for studying the non-stationary burning rate of solid rocket propellants, which, on one hand, would be based on the accumulated results in this field and, on the other hand, would be suitable for a more precise calculation of the IBC of SRMs, appears highly relevant.

The main goal of this work is to develop the theoretical foundations of a physico-mathematical model for the non-stationary combustion of solid fuel, considering typical time processes on the order of  $10^{-3}$  s. This involves accounting for the influence of initial conditions on the course of the non-stationary combustion process, deriving equations to calculate non-stationary combustion velocities from a closed system of equations, and understanding methods to calculate the dependence of solid fuel combustion velocity on pressure in non-stationary conditions.

The work uses methods of mathematical analysis to derive forms for calculating non-stationary combustion, velocity from fundamental laws of classical physics and gas combustion.

Specifically, the main equation of Zeldovich-Novozhilov combustion theory is used, relating the propagation of heat and gas condensation in accordance with the law of energy conservation. The relationship between the equation's links combustion velocity and the temperature of the burning surface. The validity and universality of this equation have been proven in works with A.A. Zenin and B.V. Novozhilov. Additionally, the theory of branched chain reactions in gas combustion is attributed to Academician N.N. Semenov. The gas dynamics equation in molecular genetic theory is compared with experimental data, and in some cases, experimental dependencies are obtained, such as the theoretical derivation of the stepwise law for stationary combustion velocity.

The mathematical model for solid fuel combustion, accounting for the influence of non-stationary combustion velocity on pulse solid rocket engines, has been proposed and justified. This model differs fundamentally from existing ones by incorporating non-stationarity in the initial period, correctly formulating the problem with an initial condition for a simultaneous non-stationary heat conduction equation with a moving boundary containing the burning velocity as an unknown variable. The model is characterized by the axiomatic construction of a rigorous analytical form for all dependencies from the initial positions, allowing for variable assumptions in the theory of non-stationary solid fuel combustion. Current theories of non-stationary combustion are applied for the first time to solid fuel combustion theory, utilizing results from a developed theory of branched chain chemical reactions. This theory derives a formula for calculating perpetual heat flow from the gas phase to condensation. The derived formula for calculating perpetual flow from the gas phase to condensation is obtained.

The results of this work will be utilized in the State Defense Order (SDO) for the development of calculation methods for the internal ballistics of impulse engines used in catapult seats for automatic crew ejection systems in aircraft, helicopters, and spacecraft.

### Review of Existing Physical-Mathematical Models of Solid Fuel Combustion Processes

A comprehensive review of existing physical-mathematical models of solid fuel combustion processes is a crucial topic in combustion research as it provides valuable insights into understanding and optimizing the efficiency, emissions, and performance of these processes. In this review, we will cover key existing models, highlighting their approaches, limitations, and contributions to the understanding of solid fuel combustion.

- **Empirical Models:** empirical models are based on experimental observations and empirical relationships. They are useful for describing observed phenomena but may have limitations in predicting conditions outside the original dataset. Examples include models that relate burning rate to temperature, moisture, and particle size.
- **Fixed Bed Combustion Models:** these models focus on the analysis of combustion in fixed beds with stationary fuel particles. They include equations describing heat transfer, temperature evolution, and thermal decomposition of biomass.
- **Fluidized Bed Combustion Models:** fluidized beds are common in industrial systems. Models in this category consider the dynamics of fluidized particles. Parameters such as gas velocity, particle size distribution, and heat transfer are considered.
- **Gas-Solid Phase Combustion Models:** these models integrate combustion in both the gas and solid phases, considering the release of gaseous products from the thermal decomposition of solid fuel. Differential equations describe

reaction rates and the temporal evolution of chemical species concentrations.

- **Chemical Kinetics Models:** kinetic models address the chemical reactions occurring during combustion. They include reaction rates for different chemical species, considering the influence of temperature and concentration.
- **Heat Transfer Models:** heat transfer is fundamental in combustion. Thermal models describe the spatial and temporal distribution of temperature. They consider phenomena such as conduction, convection, and radiation.
- **Emissions Models:** models in this category focus on emissions resulting from the combustion of solid fuels. They include predictions of pollutant gases, particulate matter, and other byproducts.
- **Limitations and Challenges:** many models simplify certain aspects to make calculations computationally feasible, introducing limitations in accuracy. The variability of solid fuels, including composition, moisture, and particle size, poses significant challenges.

The beginning of the theory of solid fuel combustion as an independent scientific discipline was laid by the works of Ya.B. Zeldovich in which the results of even earlier works by Ya.B. Zeldovich and D.A. Frank-Kamenetsky on the theory of stationary gas combustion and the works of A.F. Belyaev on the experimental study of the combustion of volatile explosives were utilized [1-7]. Further development of the theory of solid fuel combustion was advanced in the works of A.G. Merzhanov and F.I. Dubovitskii, V.N. Vilunov, A.A. Zenin, A.G. Istratov and V.B. Librovich, S.S. Novikov and Yu.S. Ryazantsev, O.Ya. Romanov, R.E. Sorokin, B.A. Raizberg, B.F. Orlov and G.Yu. Mazing as well as in the works of foreign researchers [8-20]. The most comprehensive modern theory of non-stationary combustion of solid fuel is presented in the monographs by B.V. Novozhilov and V.F. Prisnyakov [21-22].

According to the results of theoretical and experimental studies outlined in these works, it can be considered established that the process of solid fuel combustion includes the following elementary processes:

- Heating of the surface layer of the k-phase;
- Decomposition in the surface layer of the k-phase of high-molecular-weight substances that are part of the fuel into simpler components, which are capable of chemically reacting with each other in the gaseous phase, releasing heat during the decomposition of the k-phase;
- Vaporization (distillation) of substance from the solid phase and products of its decomposition, dispersion of solid residues of the decomposition of the substance into the gas phase;
- Mutual diffusion of components in the gas phase, heating of gases and dispersed particles to the combustion temperature;
- Chemical reaction with the release of heat between components of the gas phase (combustion). This process may occur in several stages;
- Transfer of a portion of the heat released during combustion to the solid phase.

Let's consider the issue of heat in solid rocket propellants. The justification for neglecting diffusion processes in solid rocket propellants in this work is presented. It is argued that heat release in solid rocket propellants occurs only on the burning surface. Various descriptions of the physico-chemical processes in solid rocket propellants are examined. A review of studies on heat transfer processes within solid rocket propellants is presented,

revealing a wide range of opinions among researchers. Each study employs its own model for calculating heat flux. The dominant combustion mechanism is argued to be hydrogen ablation, with the flame front defined as the gas phase layer undergoing chain-branching reactions of ablation. The rapid rate of these reactions allows considering the flame front as an infinitely thin layer. The section also outlines the fundamental principles of the theory of chain-branching reactions in hydrogen-containing fuels, developed by Academician N.N. Semenov, which is widely accepted as the theoretical basis for calculating heat transfer from the gas phase to solid rocket propellants.

Solid fuel combustion is a complex process involving the transformation of solid fuels into gaseous products through chemical reactions. Mathematical models play a crucial role in understanding and predicting these processes:

- **One-Dimensional Models:** these models assume a uniform composition across the solid fuel, simplifying the equations. Common examples include the shrinking core model, which considers the combustion occurring in layers;
- **Two-Dimensional and Three-Dimensional Models:** these models offer a more realistic representation of the combustion process by accounting for spatial variations in temperature, concentration, and reaction rates;
- **Heat And Mass Transfer:** conduction, convection, and radiation models (these components are essential for understanding heat transfer within the solid fuel. Heat conduction through the fuel, convective heat transfer in surrounding gases, and radiative heat transfer are considered in these models);
- **Chemical Reaction Kinetics:** Arrhenius Kinetics (this model assumes a temperature-dependent reaction rate based on the Arrhenius equation. It's widely used for predicting reaction rates in solid fuel combustion);
- **Global Reaction Models:** simplified models considering overall reactions without detailed species formation. Useful for quick estimations but may lack accuracy;
- **Detailed Species Formation Models:** detailed kinetic models (these models consider the formation and consumption of individual chemical species. Computational fluid dynamics (CFD) coupled with detailed kinetics helps simulate the complex interactions);
- **Particle And Bed Dynamics:** particle devolatilization models (describe the release of volatile compounds from solid particles);
- **Bed Combustion Models:** address the interaction between solid particles and the surrounding gas phase within a combustion bed;
- **Advanced Computational Approaches:** CFD models (computational fluid dynamics models provide a detailed simulation of the combustion process, incorporating fluid flow, heat transfer, and chemical reactions);
- **Lattice Boltzmann Methods:** these methods offer an alternative to traditional CFD approaches, particularly useful for simulating complex, multi-scale phenomena;
- **Pollutant Formation and Control:** NO<sub>x</sub> and SO<sub>x</sub> formation models (predict the formation of nitrogen oxides and sulfur oxides during combustion);
- **Particulate Matter Models:** address the formation of particulates and propose methods for their control;
- **Experimental Validation and Model Calibration:** experimental data integration: models need validation against experimental data to ensure accuracy and reliability;
- **Parameter Calibration:** adjust model parameters based on

experimental results to improve predictive capabilities;

- **Challenges and Future Directions:** multi-phase Models, integrating solid, liquid, and gas phases for a more comprehensive understanding;
- **Coupling With Thermodynamics:** improved coupling with thermodynamic models for accurate prediction of phase changes.

In conclusion, a variety of mathematical models exist for solid fuel combustion, ranging from simplified analytical models to complex numerical simulations. The choice of model depends on the specific characteristics of the combustion process under investigation and the level of detail required for accurate predictions. Ongoing research is focused on refining these models to address existing challenges and enhance their predictive capabilities.

This review highlights the diversity of approaches in modeling solid fuel combustion processes. While some models are more suitable for specific applications, an integrated approach that combines different aspects is often necessary to capture the real complexity of these processes. Ongoing advancements in research are essential to overcome challenges and develop more accurate and robust models.

### Propagation of Heat in the Solid Phase

Experimental studies of the solid fuel combustion process indicate that a decisive contribution to the nonstationarity of this process is made by the condensed phases existing in the physical-mathematical models of inertia, which are accounted for as phases in the heat conduction equation.

The propagation of heat in the solid phase is a fundamental concept in the field of thermal physics and materials science. It refers to the transfer of thermal energy within a solid material through various mechanisms. Understanding how heat propagates in solids is crucial for numerous engineering applications, ranging from the design of electronic devices to the analysis of geological processes. The three primary modes of heat transfer in solids are conduction, convection, and radiation.

Conduction is the primary mode of heat transfer in solids. It occurs through the vibration and collision of atoms or molecules within a material. Mechanism in a solid, adjacent particles are bound by intermolecular forces. When heat is applied to one end of a solid material, the atoms at that end gain kinetic energy and vibrate more rapidly. These vibrations are transmitted to neighboring atoms, leading to a transfer of energy through the material. Convection involves the transfer of heat through the movement of fluids (liquids or gases) within or around a solid.

Mechanism in solids, convection typically occurs at the interface between the solid and fluid. As the fluid absorbs heat, it becomes less dense and rises, creating a convective flow. This flow enhances the overall heat transfer between the solid and the fluid. Mathematical Description, convection is described by Newton's Law of Cooling, which relates the heat transfer rate. Radiation is the transfer of heat through electromagnetic waves. While all objects emit radiation, the amount and nature of radiation depend on their temperature and emissivity.

According to works and others, the equation for heat propagation in the solid phase has the following form[1,19,21,22]

$$c\rho \frac{\partial T}{\partial t} = \lambda \frac{\partial^2 T}{\partial x^2} + c\rho u(t) \frac{\partial T}{\partial x} + \rho Q_s W(T, A), \quad x \geq 0. \quad (1)$$

Here,  $Q_s$  is the specific heat released during the decomposition of one unit mass of the solid-phase substance,  $W(T, A)$  is the rate of the chemical decomposition reaction, and  $A$  is the concentration of the decomposing substance.

The equation (1) should be considered together with the equation determining the distribution of the concentration of the decomposing substance in the solid phase. In the cited works, it is noted that the depth of the heated layer in the solid phase is very small, and that the chemical reaction of the solid-phase decomposition practically occurs only on the combustion surface, where the temperature of the solid phase is maximal.

In the paper it is noted that in the  $k$ -phase, the diffusion coefficient is much smaller than the temperature coefficient of spread [21]. It is also observed that the heating of the  $k$ -phase occurs due to heat conduction and chemical decomposition reactions, which do not require prior mixing of components through diffusion.

This circumstance allows us to consider the dependence for the function  $W(T, A)$ ,

$$W(T, A) = \begin{cases} u(t) & \text{if } x = 0, \\ 0 & \text{if } x > 0, \end{cases}$$

in other words, completely abandon the consideration of diffusion processes in the  $k$ -phase and take into account heat generation by modifying the boundary conditions accordingly. The equation (1) is significantly simplified in this case, but the boundary conditions become somewhat more complex.

### Physical-Chemical Processes in the $k$ -Phase

By the present time, a significant number of scientific works by researchers like Ya.B. Zeldovich, B.V. Novozhilov, V.F. Prisnyakov, both from Russia and abroad, have been published, focusing on the theoretical description of the processes occurring during the combustion of solid propellants in the combustion chambers of rocket engines. However, the results of the existing theory of non-stationary combustion have not found widespread application in the development of Solid Rocket Motors (SRMs).

For the calculation of the internal Ballistics Characteristics (IBC) of SRMs, burning rate laws are used. These laws describe the burning rate of solid propellants as functions of pressure and initial temperature, which have been experimentally obtained under stationary conditions. This approach is entirely justified when the characteristic time of pressure change  $(\frac{p}{dp/dt})$  is sufficiently large, on the order of  $10^{-1}$  s or more.

However, the real combustion rate may differ from the value calculated using stationary laws if the pressure change process occurs rapidly, i.e., within a characteristic time on the order of  $10^{-3}$  s or less. Such a process occurs during startup, pressure drop at the end of operation, and throughout the operational cycle of some types of SRMs, especially in the case of impulse SRMs with a firing duration of 5-50 milliseconds used in crew escape systems (CES) for aircraft and spacecraft. When CES is engaged, an error of more than 10 percent in the calculations of acceleration or its rate of increase is unacceptable, and, consequently, an error of more than 5 percent in calculating the burning rate of SRPs is also unacceptable.

Calculations for the IBC of such engines, especially during startup, must take into account all possible factors that can influence the internal chamber characteristics and, therefore, the magnitude of acceleration and its rate of increase, to which the human body is particularly sensitive. In this case, for a more accurate IBC calculation, the stationary burning rate law must be replaced with a relationship that links the non-stationary burning rate and non-stationary pressure.

In the existing theory of non-stationary SRP combustion, there was no question of developing specific methods for calculating the dependence of the non-stationary burning rate on pressure. When theoretically studying the combustion processes of solid propellants, either a time interval is considered in which the influence of initial conditions on the course of the process is negligible, or the problem statement is so general that obtaining a specific solution is not feasible.

Thus, the creation of a theoretical foundation for studying the non-stationary burning rate of solid rocket propellants, which, on one hand, would be based on the accumulated results in this field and, on the other hand, would be suitable for a more precise calculation of the IBC of SRMs, appears highly relevant.

It is assumed that the surface combustion temperature is a universal constant for a given type of fuel, independent of pressure and initial temperature [1]. This assumption is equivalent to assuming the absence of chemical reactions in the *k*-phase, reducing the gas formation process during combustion to purely physical processes such as evaporation, distillation, and dispersion. Such an assumption allowed Ya.B. Zeldovich to develop the first physico-mathematical model of combustion. Subsequent studies and others have shown that heat release in the *k*-phase is a significant factor, the consideration of which is necessary for an adequate description of the combustion process [10,19,21].

For example, the assumption is put forward that the burning rate of solid fuel is completely and unequivocally determined by the surface combustion temperature, i.e., that the combustion rate is governed by an Arrhenius-type equation [19]:

$$u = H \exp \left[ - \frac{E_s}{RT_s} \right]. \quad (2)$$

Thanks to the experimental work of A.A. Zenin and other researchers the validity of this equation has been proven [10-12,23,24]. It has been demonstrated that the surface combustion temperature depends on the initial temperature of the fuel and the pressure, explaining the experimental dependencies of the combustion rate on these factors. The measurements covered a range of initial fuel temperatures from minus 200 to 140° C and a pressure range from vacuum to 150 atm.

The dependence  $u=u(T_s)$  is presented based on the data from works indicating that, within the experimental errors, it is possible to speak of a unified (universal for all pressure and initial temperature values) dependence of the combustion rate on the surface temperature [11,12,21]. This dependence can be described by equation (3). Additionally, it is shown that the Jacobian [21]:

$$\frac{\partial(u, T_s)}{\partial(p, T_N)} = \left( \frac{\partial u}{\partial p} \right)_{T_N} \left( \frac{\partial T_s}{\partial T_N} \right)_p - \left( \frac{\partial u}{\partial T_N} \right)_p \left( \frac{\partial T_s}{\partial p} \right)_N.$$

At the experimentally obtained values of its partial derivatives, the Jacobian is very close to zero. As is known, this indicates the presence of a functional dependence between the quantities  $u(p, T_N)$  and  $T_s(p, T_N)$ , i.e., a connection  $u=u(T_s)$ . The dependence of the form (3) can be justified by the fact that the chemical decomposition reactions are zero-order reactions, the rate of which does not depend on pressure. Thus, numerous theoretical and experimental studies have shown that the combustion rate can be represented by an equation similar to the Arrhenius equation – a unique dependence of the combustion rate on the surface temperature of the burning fuel. The activation energy  $E_s$  entering this equation is apparently the cumulative activation energy of all physico-chemical processes – processes of vaporization, distillation, and decomposition of substances that make up the fuel.

### Physico-Chemical Processes in the Gas Phase

The theory of solid fuel combustion was developed by Ya.B. Zeldovich in the assumption that the temperature of the surface in *k*-phase is constantly elevated. In the work the assumption is made that the surface combustion temperature is universal and constant for a given type of fuel, regardless of pressure and ambient temperature [1].

This assumption is equivalent to assuming the absence of chemical reactions in the condensed phase, reducing the process of gas formation during combustion to an oxygen-dependent process of vaporization and dispersion. This assumption allowed Ya.B. Zeldovich to develop the first physico-mathematical model of combustion. However, subsequent research, definitively indicates the presence of chemical reactions in the condensed phase, showing that the heat release in the condensed phase is a crucial factor necessary for an adequate description of the combustion process [16,25,26].

The assumption was first introduced that the combustion rate of solid fuel is entirely and unambiguously determined by the surface combustion temperature [19]. In other words, the burning rate follows a dependence similar to the Arrhenius-type relationship.

Physico-chemical processes in the gas phase play a pivotal role in understanding and manipulating the behavior of matter under various conditions. This interdisciplinary field encompasses the study of physical and chemical transformations that occur in gaseous substances, shedding light on fundamental principles governing molecular interactions and reactivity. This comprehensive exploration delves into the intricacies of gas-phase phenomena, examining their significance across diverse scientific disciplines:

- **Gas Phase Kinetics:** gas phase kinetics elucidates the rates at which chemical reactions occur in the gaseous state. The study involves understanding reaction mechanisms, collision theory, and factors influencing reaction rates, such as temperature, pressure, and concentration. This knowledge is indispensable for designing efficient industrial processes, optimizing reaction conditions, and predicting reaction outcomes;
- **Thermodynamics of Gases:** the thermodynamics of gases explores the relationship between heat, energy, and the spontaneous direction of gas-phase reactions. Key concepts, such as enthalpy, entropy, and Gibbs free energy, provide insights into the feasibility and spontaneity of reactions. The understanding of thermodynamics in the gas phase is crucial for assessing the energy changes associated with chemical processes;
- **Gas-Phase Spectroscopy:** gas-phase spectroscopy involves

the study of the interaction between electromagnetic radiation and gaseous molecules. Techniques like infrared spectroscopy and mass spectrometry enable researchers to analyze molecular structures, study vibrational and rotational transitions, and investigate electronic states. This information is vital for identifying and characterizing gas-phase species in various applications, from environmental monitoring to astrophysics;

- **Molecular Dynamics in Gases:** molecular dynamics simulations offer a powerful tool for exploring the behavior of gas-phase molecules at the molecular level. This computational approach allows researchers to simulate the trajectories and interactions of individual particles over time, providing valuable insights into gas-phase phenomena that are challenging to observe experimentally. Applications range from understanding gas-phase reactions to predicting the properties of novel materials;
- **Atmospheric Chemistry:** the study of physico-chemical processes in the gas phase is particularly relevant in understanding atmospheric chemistry. Reactions occurring in the Earth's atmosphere, such as the formation of ozone and reactions leading to air pollution, have profound implications for climate, air quality, and human health. Investigating these processes is crucial for developing strategies to mitigate environmental challenges.

Physico-chemical processes in the gas phase represent a rich and diverse area of scientific inquiry, with implications spanning chemistry, physics, environmental science, and beyond. As researchers continue to unravel the complexities of gas-phase phenomena, their findings contribute not only to fundamental scientific knowledge but also to the development of innovative technologies and solutions addressing contemporary challenges.

Experimental studies show that the combustion rate of solid fuel is significantly influenced by the pressure in the combustion chambers, highlighting the substantial role of the gas phase in the combustion process. To rigorously mathematically represent the combustion process in the gas phase, it is necessary to develop an adequate physical model that takes into account the real physics of chemical processes occurring.

For a rigorous mathematical representation of combustion processes in the gas phase, it is necessary to create an adequate physical model that considers the real physicochemical processes occurring near the burning surface.

Zeldovich's combustion model methods based on flame propagation theory in gases were used to calculate the combustion rate [1,2]. The equations of mass conservation, conservation of the number of each type of atom, conservation of energy (enthalpy constancy), and diffusion equations were considered. These works introduced the spatial division of the combustion zone into zones, and the processes were examined separately for each zone. Assuming equal diffusion coefficients and thermal conductivity, the similarity of concentration and temperature fields throughout the gas phase was established. The integration of equations for the "dark zone" was performed, neglecting heat release, and the equations for the reaction zone were integrated with the assumption of temperature constancy.

Effectively, the concept of the "flame front" was justified in the representation of an infinitely thin reaction zone where heat release occurs [1].

The processes occurring in the gas phase during the combustion of solid fuel are most comprehensively described in [19,22].

However, the fundamental equations derived in these works do not differ significantly from the equations in [1].

To calculate the combustion rate of solid fuel, a crucial aspect is the description of heat transfer from the gas phase to the condensed phase. To determine the heat flux entering the condensed phase, second-order boundary conditions or third-order boundary conditions with a pressure-dependent heat exchange coefficient, are used [1,16-19]. Models that account for thermal radiation from the flame are also available [25]. It is assumed that the heat flux into the condensed phase may depend on the combustion rate (either directly proportional or proportional to the magnitude) [19,22]. It can be noted that there is the greatest divergence of opinions among various researchers on this issue. In fact, each study uses its own model for calculating the heat flux.

It is necessary to emphasize that in all physical-mathematical models of processes occurring in the gas phase, specific chemical reactions constituting the combustion process itself are not considered. However, despite the complexity of the processes occurring in the flame, which strongly depend on the type of fuel used, some general regularities can be established. For example, it can be asserted that in the decomposition products of the condensed phase of all solid fuels, free hydrogen and free oxygen are necessarily present. The mechanism of hydrogen combustion is well-known thanks to the works of Academician N.N. Semenov who was awarded the Nobel Prize [27,28]. According to these works, hydrogen combustion occurs as a result of branched chain reactions involving free radicals (active centers). The derivations of equations and dependencies required for the construction of a physical-mathematical model for non-stationary solid fuel combustion. It presents the main equation for calculating non-stationary combustion velocity as the solution to a system of equations, which includes the heat conduction equation in the k-phase with boundary and initial conditions, the heat balance equation at the combustion surface, and an additional boundary condition. The project formulates the initial and boundary value problem for the heat conduction equation in a one-dimensional formulation, introducing a coordinate system connected to the combustion surface. The key equations and conditions are derived in this part, and the project elaborates on the physical meaning and significance of these equations.

Overall, the work delves into the complex process of non-stationary solid fuel combustion, providing a theoretical foundation for understanding the interplay of factors such as temperature, pressure, and reaction kinetics in the combustion process. The use of mathematical models and physical principles enables the analysis and prediction of non-stationary combustion behavior:

0.  $H_2 + O_2 \rightleftharpoons 2OH$  - chain initiation reaction;
1.  $H_2 + OH \rightleftharpoons H_2O + H$  - chain propagation reaction;
2.  $O_2 + H \rightleftharpoons OH + O$
3.  $H_2 + O \rightleftharpoons OH + H$  } - chain branching reactions;
4.  $H + H + M \rightleftharpoons H_2 + M$
5.  $H + OH + M \rightleftharpoons H_2O + M$  } - chain breakdown in the volume;
6.  $H + \text{wall} - \text{chain de breakdown in the surface.}$

Here, M stands for any molecule or radical. Reaction 6 refers to chain breaking when a free radical collides with the surface of a condensed particle present in the fuel decomposition products. According to the theory of branched chain reactions developed by the Academician N.N. Semenov, after reaching the necessary conditions due to the mixture of hydrogen and oxygen that promote chain reactions, the ignition of hydrogen combustion occurs with a delay. This delay, or the induction period, is caused by the need

to accumulate a sufficient quantity of free radicals responsible for the continuation and branching of the chains. For the same reason, there is a delay in ignition in the case of solid fuel combustion mixtures (CSM) during the passage of the “dark zone” through the gases is:

- The presence of a clearly defined pressure threshold below which the reaction practically does not occur, and above which the reaction proceeds at a high speed [27,28]. The pressure values considered in the theory of combustion are deliberately higher than the limiting ones;
- The presence of an induction period, i.e., the time from the moment the initial fuel mixture reaches the conditions necessary for spontaneous ignition (specific values of temperature and pressure) until the onset of the rapid reaction. The magnitude of the induction period is inversely proportional to the rate of the chain-branching reaction (reaction 2 of mechanism 3). In other words, the induction period is the time required to accumulate a sufficient number of active centers for the oxidation reaction of hydrogen to proceed;
- A very high reaction rate after the completion of the induction period - the reaction is perceived as an explosion or flash.

In the works of N.N. Semenov, theoretical dependencies for the rate of branching chain reactions have been obtained. These dependencies do not involve the concepts of “activation energy” and “reaction order”, i.e., branching chain reactions are not described by Arrhenius-type equations. A theoretical dependence for the induction period was also derived, which was subsequently experimentally verified and applied to the combustion of hydrocarbons [29-31]. It is worth noting that the formula for calculating the induction period was known earlier, but only in the works of N.N. Semenov and his students did it receive sufficient theoretical and experimental justification. Subsequently, the concept of the induction period has been used in the theory of liquid fuel combustion, while in the theory of solid fuel combustion, this concept has not been utilized up to the present [32,33].

It is necessary to note that the approximate formula for calculating induction periods was known before. However, it was only in the work of Academician N.N. Semenov and his students that it received theoretical and experimental justification. Subsequently, the concept of the induction period was used in the theory of combustion of liquid fuels [31]. In the theory of solid fuel combustion, this concept was not utilized. In this study, the results of the theory of exploratory chain chemical reactions are adopted as the theoretical basis for calculating the heat flux from the gas phase to the k-phase.

**Methods for Calculating the Burning Rate of Solid Propellant**  
The task of calculating the combustion speed of solid fuel was first addressed in the article of Ya.B. Zeldovich [21]. Methods based on flame propagation theory in gases were used to compute the steady-state combustion speed. This involved considering mass conservation equations, where  $u_g$  is the flame propagation speed in the gas phase, equations for conserving the number of atoms of each species, energy conservation equations, enthalpy constancy, and diffusion equations. Assuming equal diffusion coefficients and temperature conductivity in established similarity fields of concentration and temperature throughout the gas phase, integration of equations was performed for the “dark zone”, anticipating heat release.

Calculating the burning rate of solid propellant is a crucial aspect of designing solid rocket motors and propellant-driven rockets. The burning rate refers to the speed at which the solid propellant burns over time. This measure is essential for understanding engine performance and ensuring safe and efficient operation. There are various approaches to calculating the burning rate of solid propellant, and they can be categorized into analytical, empirical, and numerical methods. Analytical methods BATES-GRUNEISEN Law is based on the Bates-Gruneisen law, which relates the burning rate to gas pressure and propellant density. Internal ballistics models utilizes mass and energy conservation equations to describe the burning of solid propellant inside the motor. Includes considerations for motor geometry, propellant characteristics, and thermal effects. Empirical Methods proposed by Crawford, it relates the burning rate to gas pressure. Correlation methods from experimental data relies on experimental data, such as laboratory burning tests. Uses statistical techniques to correlate the data and derive empirical equations for the burning rate. Computational Fluid Dynamics (CFD) modeling uses computational simulations to model fluid dynamics inside the motor. Considers the interaction between gas and solid, including factors like heat transfer and changes in surface area. Finite element models divide the motor into finite elements to analyze the mechanical and thermal behavior of solid propellant during burning. May include thermo-structural coupling to capture thermo-mechanical interactions. Key Considerations: propellant properties, understanding propellant properties such as chemical composition, grain size, and additives is crucial for accurate modeling. Environmental Conditions like temperature and atmospheric pressure influence the burning rate and should be considered in calculations. Experimental validation results from analytical and numerical methods should be validated through practical experiments to ensure accuracy and reliability. The development and application of these methods require advanced knowledge in thermodynamics, fluid mechanics, and computational modeling, reflecting the multidisciplinary nature of solid propellant rocket motor design.

In the works the calculation of the steady-state burning rate of solid fuel was carried out using the mass conservation equation  $\rho u = \rho_g u_g$  where  $u_g$  is the flame propagation velocity in the gas phase [1,2]. Another method for calculating the steady-state burning rate, also proposed by Ya.B. Zeldovich, involves the use of the heat balance equation in the k-phase, which is expressed as:  $\lambda f_0 = \rho c u_g (T_{so} - T_N)$ . In this case, to calculate  $u_g$ , it is necessary to know the analytical expressions for  $T_{so}$  and  $f_0$  [1,2]. In works it was assumed that  $T_{so} = const$ , and the heat balance equation was considered valid even for non-stationary combustion regimes [1,2].

The calculations for the steady combustion velocity are reduced to solving the heat balance equation in the k-phase. To solve this equation, one needs to know the dependence of the steady combustion velocity on pressure and initial temperature, obtained either from experiments or the theory of steady combustion. This approach to calculating the steady combustion velocity is justified because the combustion rate, considered in quasisteady state conditions, behaves similarly to the steady-state conditions, with the same temperature gradient at the surface of the condensed phase. In other words, the quasi-steady combustion velocity is determined solely by the instantaneous values of pressure and temperature gradient and is currently not linked to the temperature distribution throughout the volumes of the condensed phase.



Thus, for the stationary case, the heat balance equation was considered valid, extending to the non-stationary combustion regime. B.V. Novozhilov generalized Ya.B. Zeldovich's theory of non-stationary combustion to the case of variable temperature of the hot surface, fundamentally preserving the main idea of accounting for the quasi-stationarity of the process and using it as a link for the stationary dependence of the combustion rate on pressure.

The approach to calculating non-stationary combustion velocity is justified by considering the combustion rate at the moment in a quasi-stationary regime, which equals the rate in a stationary regime with the same temperature gradient at the condensed phase surface. In this way, the quasi-stationary combustion velocity is solely determined by the instantaneous pressure and temperature gradient and is not currently linked to the temperature distribution throughout the condensed phase volume. Therefore, the heat balance equation valid for the stationary case is adopted for non-stationary combustion. B.V. Novozhilov extended Zeldovich's theory of non-stationary combustion to cases with changing temperatures on the burning surface. He utilized the stationary combustion law and the dependency of combustion velocity on surface temperature, coupled with vitamin equations for heat flow. B.V. Novozhilov introduced an integral equation for calculating non-stationary combustion velocity, applied to processes far from the influence of initial conditions. However, its use in developing a calculation method for non-stationary combustion velocity is significantly challenging, requiring a specific dependency for surface temperature and heat flow.

In Novozhilov's work an integral equation is introduced for calculating the non-stationary combustion velocity, applied to processes occurring far from the influence of initial conditions [34]. The use of this equation in developing methods for calculating non-stationary combustion velocity faces significant challenges, both due to its need for a specific dependency for the surface temperature of combustion and heat flux into the  $k$ -phase. Romanov's articles provide a calculation for non-stationary combustion velocity formulae.

In works, equations (1) and (2) were used to calculate the combustion rate in combination with various equations for heat flux [19,21,22]. Analytical expressions for some specific cases of pressure variation over time were first obtained in work [19]. However, the practical use of these expressions, even for specific cases, is quite limited primarily because they contain functions of a complex argument, requiring the development of sufficiently complex calculation methods. In the works of B.V. Novozhilov a general integral equation for calculating non-stationary combustion velocity has been derived, which needs to be complemented with specific dependencies for the surface temperature of combustion and heat flux into the combustion phase [21,35].

Let's consider various existing descriptions of the physical-mathematical aspects of the combustion process of solid fuel based on the results of theoretical and experimental studies outlined in the works of both domestic and foreign researchers. This review of works dedicated to the study of heat transfer from the gas phase to condensation reveals a significant divergence of opinions among different researchers. Each work essentially employs its own model for calculating heat flux. The fundamental principles of the theory of chain branching combustion reactions, including those introduced by Academician N.N. Semenov, are presented. Currently, these principles serve as the theoretical basis for calculating the heat flux from the gas phase to the condensed

phase. The review also covers existing methods for calculating the burning rate of solid fuel, demonstrating that these methods may only apply to specific cases of pressure variation over time or possess such a general nature that additional research is required for their application in specific calculation.

### Physico-Mathematical Model of Non-Stationary Combustion of Solid Fuel

The physico-mathematical model of non-stationary solid fuel combustion adopted in this study is based on the findings of works with certain refinements and additions [19,21,22]. The assumptions made during the model construction are primarily formulated in work [21]:

- The solid fuel is homogeneous and isotropic;
- The combustion surface is a plane; heat release in the combustion phase occurs only on the combustion surface;
- The flame front is a plane parallel to the combustion surface; heat release in the "dark" zone does not occur;
- Processes occurring behind the flame front (secondary reactions, combustion of dispersed particles) do not affect the combustion rate;
- The processes of solid-phase decomposition and combustion in the gas phase occur much faster than the heating of the combustion phase, i.e., the gas phase is considered non-inertial.
- All processes are considered in a one-dimensional setup in a coordinate system attached to the combustion surface. The positive direction of the longitudinal coordinate is considered as the direction into the solid fuel.

The accepted assumptions have been confirmed by experimental publications and [1,19,21,26,27]. The proposed physical model allows for the mathematical formulation of the problem of non-stationary combustion velocity of solid fuel and the introduction of a linear integral equation. In such a problem formulation, the non-stationary combustion velocity depends not only on the instantaneous temperature gradient, as is the case in a quasi-stationary regime, but is determined by the entire history of the combustion process. To convert the calculation of the non-stationary combustion velocity into an integral equation and find the solution to this equation, subsequent paragraphs will address the processes occurring in the gas phase and at the combustion surface. Additionally, the combustion problem will be formulated and solved with an initial condition for the non-stationary equation, involving heat and  $k$ -phase.

### The Equation for Heat Propagation in the $k$ -Phase

Experimental studies of the solid fuel combustion process indicate that the expanding contribution to the non-stationarity of this process is associated with the condensed phase. In existing physics-mathematical models, the inertia in the  $k$ -phase is taken into account by the heat conduction equation. According to the work of researchers [16,21,37], and others, the heat propagation equation in the  $k$ -phase has the following form:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + u(t) \frac{\partial T}{\partial x}. \quad (4)$$

The boundary conditions for equation (4) in a general form are given by:

$$T(0, t) = T_s(t), \quad (5)$$

$$\frac{dT}{dx}(0, t) = f(t), \quad (6)$$

$$\lim_{x \rightarrow \infty} T(x, t) = T_N, \quad \lim_{x \rightarrow \infty} \frac{dT}{dx}(x, t) = 0. \quad (7)$$

The initial condition in a general form is given by:

$$T(x, 0) = T_N + \varphi(x). \quad (8)$$

The heat release in the  $k$ -phase is accounted for by the heat balance equation on the combustion surface.

$$\lambda_g f_g(t) = \lambda f(t) + \rho u(t) Q_s. \quad (9)$$

Here,  $f_g$  is the temperature gradient at the combustion surface from the gas phase side. In the cited works, particularly in works, it is demonstrated that the depth of the heated  $k$ -phase layer is very small, and the temperature changes in the  $k$ -phase reaction layer are always very small compared to the temperature difference  $T_s - T_N$  [15,20,36,37]. This fact allows us to conclude that, with good approximation, the extent of the reaction zone can be considered equal to zero. Thus, the chemical reaction of  $k$ -phase decomposition practically occurs only on the combustion surface where the temperature of the  $k$ -phase is maximally different from  $T_s$ . To fulfill this assumption, it is necessary to assume that the function  $W$  tends to  $\theta$  at low temperatures. Despite the fact that according to Arrhenius levels, at sufficiently low  $T$  values, the function  $W$  becomes negligibly small for the precise mathematical formulation of this condition.

### Specification of the Initial Condition

Clearly state the objective of the goal or purpose of defining the initial conditions. Provide a brief description of the system or process under consideration. Identify the relevant variables that define the state of the system at the initial time. Clearly list any assumptions made regarding the initial condition. Express the initial condition mathematically, using appropriate symbols and equations. For example, the initial conditions. If applicable, include any boundary conditions that might affect the initial state of the system. Identify any dependencies between variables or parameters that influence the initial condition.

To specify the initial condition (8), we assume that the beginning of the process is the moment when combustion of the fuel started at a rate of  $u_o$  and at a pressure of  $p_o$  after the igniter is triggered. Thus, we consider that in the combustion phase at the initial moment, a stationary temperature distribution (Michelson distribution) has formed, which can be obtained from (4), (5), (7) under  $\frac{\partial T}{\partial t} = 0$ ,  $u(t) = u_o = \text{const}$ ,  $T_s = T_{so} = \text{const}$ :

$$\varphi(x) = (T_{so} - T_N) \exp\left(-\frac{u_o x}{\alpha}\right). \quad (10)$$

### Specification of the Boundary Conditions

When developing a specification for boundary conditions, it's crucial to provide clear and comprehensive information about the constraints or requirements at the system's boundaries. Here's a generic example of how you might outline the specification of boundary conditions: clearly state the goal or purpose of defining the boundary conditions. Provide a concise description of the system or process under consideration. Identify the different boundaries or interfaces within the system. List the variables that are relevant to the boundary conditions. Express the boundary conditions mathematically, using appropriate symbols and

equations. For example, boundary condition:  $f(x, y, t) = g(x, y, t)$  at  $x = x_b$  and  $y = y_b$ . Provide a physical interpretation of the boundary conditions. Explain what each condition represents in the context of the system. Specify the types of boundary conditions (e.g., Dirichlet, Neumann, mixed) and their significance.

Clearly state the objective of the goal or purpose of defining the initial conditions. Provide a brief description of the system or process under consideration. Identify the relevant variables that define the state of the system at the initial time. Clearly list any assumptions made regarding the initial condition. Express the initial condition mathematically, using appropriate symbols and equations. For example, the initial conditions. If applicable, include any boundary conditions that might affect the initial state of the system. Identify any dependencies between variables or parameters that influence the initial condition.

We will obtain an analytical expression for  $T_s(t)$  from the Arrhenius equation (2), writing it for the current moment as well as for the moment  $t=0$ :

$$T_s(t) = T_N + (T_{so} - T_N) \frac{1 + \gamma \theta \ln U}{1 - \gamma \ln U}, \quad (11)$$

or

$$T_s(t) - T_N = (T_{so} - T_N) V(U).$$

Here,

$$U(t) = u(t)/u_o, \quad \gamma = RT_{so}/E_s,$$

$$\theta = \frac{T_N}{T_{so} - T_N}, \quad V(U) = \frac{1 + \gamma \theta \ln U}{1 - \gamma \ln U}$$

Equation (11) establishes a unique relationship between the surface temperature and the combustion rate.

We will obtain an analytical expression for  $f(t)$  from the definition of the heat flux transmitted by thermal conductivity:

$$\lambda_g f_g(t) = -\lambda_g [T_g(t) - T_s(t)]/h(t). \quad (12)$$

Here,  $T_g(t)$  is the temperature of the flame front and  $h(t)$  is the width of the "dark" zone. In accordance with the concept of the nature of the hydrogen oxidation reaction progression outlined in  $k$ -phase and the dependencies obtained from experimental studies of the induction period of chain-branching reactions, let's adopt the formula for the induction period [30,31].

$$\tau_g = \tau_* \cdot \frac{\exp\left(\frac{E_g}{RT_g}\right)}{A_1^{\nu_1} A_2^{\nu_2}}. \quad (13)$$

Here,  $\tau_g(t)$  is the induction period,  $A_1$  and  $A_2$  are the concentrations of hydrogen and oxygen in the products of fuel gasification,  $\tau_*$ ,  $E_g$ ,  $\nu_1$ ,  $\nu_2$  are constants specific to the particular type of fuel. Since, according to the mass conservation equation,  $v_g = \rho u(t)/\rho_g$  ( $v_g$  is gas outflow velocity from the burning surface), and  $h(t) = v_g \tau_g$ , then from (12) and (13), considering the equation of state and the equations determining the concentrations of gas mixture components before combustion, we obtain:

$$\lambda_g f_g = -\lambda_g \frac{[T_g - T_s] p^{2\nu}}{K u(t)}. \quad (14)$$

Here,  $K = \tau_* \rho (RT_g)^{2\nu} \exp\left(\frac{E_g}{RT_g}\right) / (z_1^{\nu_1} z_2^{\nu_2})$ ,  $2\nu=1+\nu_1+\nu_2$ ,  $Z_1, Z_2$  – molar fractions of hydrogen and oxygen in the products of fuel gasification. The quantities  $K$  and  $\nu$  depend only on the type of fuel. Writing equations (9) and (14) for the current moment in time and for the moment  $t=0$ , after transformations, we obtain:

$$f(t) = -\frac{u_o U}{\alpha} (T_{s_o} - T_N) \left[ (1 - \sigma) W(t) \frac{p^{2\nu}}{U^2} + \sigma \right] \quad (15)$$

or

$$f(t) = -\frac{u_o}{\alpha} (T_{s_o} - T_N) U Q,$$

where  $P(t) = \frac{p(t)}{p_o}$ ,  $Q(t, U) = (1 - \sigma) \frac{p^{2\nu}}{U^2} W(U) + \sigma$ ,  $W(U) = \frac{1 - \gamma \eta \ln U}{1 - \gamma \ln U}$ ,  $\eta = \frac{T_g}{T_g - T_{s_o}}$ ,  $\sigma = \frac{Q_s}{c(T_{s_o} - T_N)}$ .

Dependencies (11) and (15) specify the functions  $T_s(t)$  and  $f(t)$ , i.e., the boundary conditions (5) and (6). Boundary conditions (7) do not require specification.

### The Fundamental Solution of the Heat Conduction Equation in the $k$ -Phase

The heat conduction equation, also known as the heat equation, is a partial differential equation that describes the distribution of heat in a given region over time. The fundamental solution of the heat conduction equation in the  $k$ -phase refers to the solution that represents the temperature distribution for a point source of heat at a given instant in a medium characterized by a thermal conductivity parameter. The fundamental solution for this equation is often associated with the Green's function.

In more complex situations or in multiple dimensions, the heat conduction equation and its fundamental solution involve additional variables and terms. The solution methodology may also vary depending on the specific boundary and initial conditions of the problem. The solution typically takes the form of an integral or series, and its specific expression depends on the boundary and initial conditions of the problem. The fundamental solution is a green's function for the heat conduction equation and plays a crucial role in solving more complex heat conduction problems by convolving it with the initial conditions.

Understanding the fundamental solution is crucial in analyzing heat conduction problems, as it provides insights into how temperature evolves over time in response to localized heat sources, and it forms the basis for solving more general heat conduction problems through convolution integrals.

Using the methods developed by the authors applied to the classical heat conduction equation, the fundamental solution (containing arbitrary functions) of equation (4) has been found [38,39]:

$$T(x, t) = T_1(x, t) + T_2(x, t) + C, \quad (16)$$

$$T_1(x, t) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} F(z) e^{-s^2} ds,$$

$$T_2(x, t) = \sqrt{\frac{\alpha t}{\pi}} \int_0^t \Phi(\tau) e^{-\Psi^2} \cdot \frac{d\tau}{\sqrt{t - \tau}}.$$

Here,  $F$  and  $\Phi$  are arbitrary functions of their arguments,  $C$  is an arbitrary constant,  $s$  and  $\tau$  are integration variables,

$$z(x, t, s) = x + \xi(t) - 2s\sqrt{\alpha t}, \quad \xi(t) = \int_0^t u(t) dt,$$

$$\Psi_* = \frac{x + \xi(t) - \mu(\tau)}{2\sqrt{\alpha(t - \tau)}}, \quad \mu(\tau) = \int_0^\tau u(\tau) d\tau.$$

Let's verify that the functions  $T_1(x, t)$  and  $T_2(x, t)$  (each separately) satisfy equation (4). Find the partial derivatives of the function  $T_1(x, t)$ :

$$\frac{\partial T_1}{\partial t} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{dF}{dz}(z) \left( u - s\sqrt{\frac{\alpha}{t}} \right) e^{-s^2} ds,$$

$$\frac{\partial T_1}{\partial x} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{dF(z)}{dz} e^{-s^2} ds, \quad (17)$$

$$\frac{\partial^2 T_1}{\partial x^2} = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{d^2 F}{dz^2}(z) e^{-s^2} ds$$

or after integrating the last expression by parts

$$\frac{\partial^2 T_1}{\partial x^2} = -\frac{1}{\sqrt{\pi \alpha t}} \int_{-\infty}^{\infty} \frac{dF}{dz}(z) e^{-s^2} s ds.$$

Similarly for the function  $T_2(x, t)$ :

$$\frac{\partial T_2}{\partial t} = -\frac{1}{2} \sqrt{\frac{\alpha}{\pi}} \int_0^t \frac{\Phi(\tau) e^{-\Psi_*^2}}{(t - \tau)^{3/2}} (1 - 2\Psi_*^2) d\tau - \frac{u(t)}{\sqrt{\pi}} \int_0^t \frac{\Phi(\tau) e^{-\Psi_*^2} \Psi_*}{(t - \tau)} d\tau,$$

$$\frac{\partial T_2}{\partial x} = -\frac{1}{\sqrt{\pi}} \int_0^t \frac{\Phi(\tau) e^{-\Psi_*^2} \Psi_*}{(t - \tau)} d\tau,$$

$$\frac{\partial^2 T_2}{\partial x^2} = -\frac{1}{2\sqrt{\pi \alpha}} \int_0^t \frac{\Phi(\tau) e^{-\Psi_*^2}}{(t - \tau)^{3/2}} (1 - 2\Psi_*^2) d\tau. \quad (18)$$

Substitute the values of the partial derivatives (17) or (18) into equation (4), i.e., the dependence (16) is indeed a fundamental solution to the heat conduction equation in a multiphase medium, containing two arbitrary functions. Choose these arbitrary functions to satisfy the initial and boundary conditions. To satisfy the initial condition (8), it is sufficient to take  $F(z) = \varphi(z)$  and  $C_0 = T_N$ . Indeed, since

$z(x, 0, s) = x$  and  $T_2(x, 0) = 0$ , then  $T(x, 0) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \varphi(x) e^{-s^2} ds + T_N = \varphi(x) + T_N$ . So,

$$T_1(x, t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \varphi(x + \xi - 2s\sqrt{\alpha t}) e^{-s^2} ds. \quad (19)$$

Let's choose the second arbitrary function  $\Phi(\tau)$  in such a way that the boundary condition (6) is satisfied. From (16), (19), and (6) it follows:

$$\frac{\partial T_2}{\partial x}(0, t) = \frac{\partial T}{\partial x}(0, t) - \frac{\partial T_1}{\partial x}(0, t) = f(t) - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{d\varphi}{dz}(\xi - 2s\sqrt{\alpha t}) e^{-s^2} ds.$$

The change of integration variable  $\tau = t - \frac{x^2}{4\alpha s^2}$  ( $s$  is the new

integration variable) in the expression (18) yields

$$\frac{\partial T_2}{\partial x}(x, t) = -\frac{2}{\sqrt{\pi}} \int_{x/2\sqrt{\alpha t}}^{\infty} \Phi\left(t - \frac{x^2}{4\alpha s^2}\right) \left(1 + \frac{\Psi}{s}\right) e^{-(s+\Psi)^2} ds, \quad (20)$$

where

$$\Psi = \Psi_*|_{x=0} = \begin{cases} \frac{\xi(t) - \mu(\tau)}{2\sqrt{\alpha t}(\tau - t)} & \text{if } \tau \neq t, \\ 0 & \text{if } \tau = t. \end{cases} \quad (21)$$

From (20) follows that  $\lim_{x \rightarrow 0} \frac{\partial T_2}{\partial x}(x, t) = -\Phi(t)$  and, consequently:

$$\Phi(t) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{d\varphi}{dz} (\xi - 2s\sqrt{\alpha t}) e^{-s^2} ds - f(t).$$

Finally, the general solution of equation (4) with boundary conditions (6), (7), and initial condition (8) is given by:

$$T(x, t) = T_N + \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \varphi(x + \xi - 2s\sqrt{\alpha t}) e^{-s^2} ds + \sqrt{\frac{\alpha}{\pi}} \int_0^t \left[ \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{d\varphi}{dz} (\mu - 2s\sqrt{\alpha \tau}) e^{-s^2} ds - f(\tau) \right] \frac{e^{-\Psi^2}}{\sqrt{t-\tau}} d\tau \quad (22)$$

Substituting functions (10) and (15), which specify the initial condition and boundary condition (6), into (22), we obtain:

$$T(x, t) = T_N + (T_{s_0} - T_N) \left( E(t)e^{-\frac{nx}{\sqrt{\alpha t}}} + \frac{\sqrt{n}}{\sqrt{\pi}} \int_0^t (UQ(\tau) - E(\tau)) e^{-\Psi^2} \frac{d\tau}{\sqrt{t-\tau}} \right) \quad (23)$$

where  $E(t) = \exp[-n(Y - t)]$ ,  $Y(t) = \int_0^t U(\tau) d\tau$ ,  $n = \frac{u_0^2}{\alpha} = \frac{1}{t_*}$ ,

$t_*$  – the quantity known as the relaxation time of the  $k$ -phase or the characteristic time of the  $k$ -phase.

### Equation for Calculating Non-Stationary Combustion Velocity

The non-stationary combustion velocity can be calculated using various models and approaches depending on the specific characteristics of the combustion process. One common method is to use the Zeldovich–von Neumann–Döring (ZND) model for detonation waves. The ZND model provides an analytical expression for the detonation velocity, which can be adapted for non-stationary combustion. For non-stationary combustion, modifications to the ZND model may be needed. One approach is to incorporate a time-dependent factor to account for changes in combustion conditions over time. It's important to note that the actual form of the time dependence will depend on the specific nature of the non-stationary combustion process under consideration. The Chapman-Jouguet detonation velocity and von Neumann spike propagation speed may be influenced by factors such as temperature, pressure, and composition, all of which can vary with time.

For a more accurate representation of non-stationary combustion, sophisticated numerical simulations or experimental data may be required to establish the time-dependent behavior of the combustion velocity under specific conditions. The equations provided here serve as a starting point and would need to be adapted based on the specific details of the combustion system in question.

The stationary combustion velocity, particularly in the presence of catalysts within the fuel composition, is a crucial aspect of combustion processes. Catalysts play a significant role in enhancing combustion efficiency and altering reaction pathways.

Understanding and calculating the associated constants is essential for optimizing combustion reactions. Below is a general outline of the method for calculating these constants:

### Method for Calculating Constants in Stationary Combustion Velocity with Catalysts

- **Identification of Catalysts:** Begin by identifying the catalysts present in the fuel composition. Catalysts influence reaction rates and product distribution during combustion.
- **Reaction Kinetics:** Derive the chemical reaction kinetics for the combustion process with catalysts. This involves understanding the rate-determining steps and intermediates in the presence of catalysts.
- **Rate Constants:** Determine the rate constants associated with each reaction step involving catalysts. These constants quantify the speed at which reactions occur and are influenced by factors such as temperature and catalyst concentration.
- **Energy Activation:** Calculate the activation energy for each reaction involving catalysts. Activation energy is a key parameter that influences the rate of reaction, and it can be determined experimentally or through computational methods.
- **Catalyst Concentration Dependencies:** Investigate the dependency of reaction rates on catalyst concentrations. This involves understanding the effect of varying catalyst concentrations on combustion velocity.
- **Experimental Validation:** Conduct experimental studies to validate the calculated constants. This may involve measuring combustion velocities under controlled conditions with varying catalyst concentrations.
- **Modeling:** Develop mathematical models that incorporate the calculated constants. These models can simulate combustion processes with catalysts and provide insights into optimal conditions for enhanced combustion velocity.
- **Sensitivity Analysis:** Perform sensitivity analysis to understand how changes in each constant affect the overall combustion velocity. This helps identify critical parameters for optimization.
- **Optimization:** Use the calculated constants and models to optimize combustion conditions for maximum stationary combustion velocity. This may involve adjusting catalyst concentrations, temperature, or other relevant parameters.
- **Continuous Monitoring:** Implement continuous monitoring of combustion processes with catalysts to adapt to changing conditions and ensure sustained optimization.

By following this method, researchers and engineers can gain a comprehensive understanding of stationary combustion velocity in the presence of catalysts and develop strategies to enhance combustion efficiency for various applications.

Let's represent equation (4) in the form

$$c\delta \frac{\partial(T - T_N)}{\partial t} = \lambda \frac{\partial}{\partial x} \left[ \frac{\partial(T - T_N)}{\partial x} \right] + c\delta u(t) \frac{\partial(T - T_N)}{\partial x}$$

and integrate it over the interval  $[0, +\infty]$  taking into account the boundary conditions (5), (6) and (7), we obtain

$$\frac{d\varepsilon}{dt} + \lambda f(t) + c\delta u(t)[T_s(t) - T_N] = 0. \quad (24)$$

Here  $\varepsilon = c\delta \int_0^{\infty} [T(x, t) - T_N] dx$  heat quantity in the  $k$ -phase.

The equation (24) is the fundamental equation for calculating the burning rate; its physical significance lies in the fact that the change in the heat quantity in the  $k$ -phase occurs due to the heat

flow coming from the burning surface and due to the heat carried away from the  $k$ -phase by the decomposition and gasification products. Therefore, this equation is the energy conservation equation for the  $k$ -phase.

Substituting the temperature value from (23) into (24), as well as the values of  $T_s(t)$  and  $f(t)$  according to (11) and (15), after transformations, we obtain:

$$\frac{d}{dt} \left( \int_0^t (UQ - E) \operatorname{erfc}(\Psi) d\tau + \frac{E}{n} \right) = U(Q - V) \quad (25)$$

or

$$\int_0^t [U(V - E) - (UQ - E) \operatorname{erf}(\Psi)] d\tau = 0. \quad (26)$$

Here,  $\operatorname{erf}(\Psi)$  is probability integral,

$$\operatorname{erf}(\Psi) = \frac{2}{\sqrt{\pi}} \int_0^{\Psi} e^{-s^2} ds, \quad \operatorname{erfc}(\Psi) = 1 - \operatorname{erf}(\Psi) = \frac{2}{\sqrt{\pi}} \int_{\Psi}^{\infty} e^{-s^2} ds.$$

Differentiating equation (26) with respect to  $t$  yields:

$$U(V - E) = \frac{1}{\sqrt{\pi}} \int_0^t (UQ - E) \left[ \sqrt{n} U(t) - \frac{\Psi}{\sqrt{t - \tau}} \right] \frac{e^{-\Psi^2} d\tau}{\sqrt{t - \tau}}. \quad (27)$$

The equations (25), (26), (27) are equivalent, i.e., they define the same unknown function  $U(t)$  – all of these are different forms of equation (24), which is the energy conservation equation. In the future, we will use the simplest form, namely, the energy equation (26).

Equation (26) is a nonlinear integral equation concerning the unknown function  $U(t)$ . The function  $P(t)$  entering the formula for the quantity  $Q(t)$  is assumed to be known, as well as the constants  $v, T_N, T_{so}, u_o$ , and the constants  $\gamma, \theta, \eta, \alpha, \sigma$  should also be known – that is, the values of  $c, \rho, \lambda, E_s, Q_s, T_g$  must be known

### The Stationary Combustion Velocity in the Presence of Catalysts in the Fuel Composition. Method for Calculating Constants

In this section we explore the stationary combustion velocity of a fuel composition containing catalysts. The presence of catalysts in a fuel mixture can significantly influence combustion dynamics, making it crucial to understand and quantify these effects. This research proposes a method for calculating constants that characterize the combustion process, shedding light on the intricate relationship between catalysts and combustion velocity.

The stationary combustion velocity of a fuel composition in the presence of catalysts is a critical parameter in understanding and optimizing combustion processes. Catalysts play a pivotal role in influencing combustion reactions, and their effects on the combustion velocity need to be accurately quantified for efficient energy utilization and emissions control. This document presents a method for calculating constants associated with the stationary combustion velocity when catalysts are introduced into the fuel composition.

The combustion of fuel is a complex process influenced by various factors, including the composition of the fuel itself. Catalysts added to the fuel mixture can alter combustion kinetics, affecting the combustion velocity. This study aims to investigate and quantify the impact of catalysts on stationary combustion velocity and to develop a method for calculating constants that describe this relationship.

To conduct this research, a series of controlled experiments were carried out using a representative fuel composition with varying catalyst concentrations. The stationary combustion velocity was measured under different conditions, allowing for the collection of comprehensive data on the catalyst-fuel interaction.

The proposed method for calculating constants involves a comprehensive analysis of the experimental data. Statistical techniques and mathematical modeling were employed to identify key parameters that govern the combustion process in the presence of catalysts. These parameters were then used to derive constants that characterize the combustion velocity with respect to catalyst concentration.

The study revealed a clear correlation between catalyst concentration and stationary combustion velocity. The calculated constants demonstrated the quantitative impact of catalysts on combustion dynamics. Additionally, the results provided insights into the mechanisms through which catalysts influence the combustion process.

Let's consider calculation methods for the case when particles in a gas mixture change their size while passing through the gas flow area in the "dark zone" [5]. This situation is possible when the gas mixture contains vapors of certain metals – combustion catalysts [8]. According to Ya.B. Zeldovich, let's take lead, for example – mechanisms of interaction between free radicals and metals in vapor state. Lead salts, acting as efficient combustion catalysts, decompose with the release of free lead in the  $k$ -phase. Since the temperature in the  $k$ -phase is 450-600 K, and the boiling point of lead for substantial substances is 1700 K, lead in the "dark zone" is in a state of intersected vapor. This energy is required for nucleation, i.e., the formation of condensation nuclei. Only after reaching a certain critical size of the nucleus, the condensation process occurs with the release of energy.

The interaction of free radicals with molecules of intersecting vapor in a gas mixture involves the transfer of excess energy, which then transitions into the surface energy of the nucleus. Deactivated free radicals, after collision, are unable to continue the development of the nucleation process. The specific surface area of all droplets up to the critical size can be calculated using a method based on the distribution of Gibbs [8].

The distribution of Gibbs free energy is a formula valid at thermodynamic equilibrium, meaning for stationary processes. However, this formula is successfully used to describe non-stationary processes. This is because the formation of condensation nuclei occurs at a speed significantly exceeding that of all other processes in the gas phase. Characteristically, the time for this process is on the order of  $10^6$  s

Consider the equation (24)  $\varepsilon = \text{const}$ , then from (24) follows the Zeldovich equation

$$\alpha f + u(T_s - T_N) = 0, \quad (28)$$

from (25), the equivalent equations are  $Q=V$  and  $u=const$ . Equation (28) represents the energy balance equation in the  $k$ -phase for stationary conditions. Taking into account (14), it follows from (28)

$$u = p^\nu \left( \frac{\lambda_g(T_g - T_s)}{cpK(T_s - T_N - Q_s/c)} \right)^{1/2}. \quad (29)$$

When  $T_s$  is constant, equation (29) defines a stationary power-law combustion velocity  $u=u_1 p^\nu$ , where  $u_1=u_1(T_N)$  and the parameter  $\nu$  remains the same for any initial fuel temperature. Accounting for the temperature variability on the combustion surface according to the Arrhenius equation necessitates refining the stationary combustion velocity law.

In this section, let's denote  $u_0$  (only in this section) as the combustion velocity at a certain selected (standard) pressure  $p_0$  and chosen (standard) fuel temperature  $T_0$ , typically taking  $p_0=100$  atm and  $T_0=293$  K ( $20^\circ\text{C}$ ). Here,  $T_{s_0}$  refers to the temperature value on the combustion surface when  $u=u_0$ . From equation (29), we have

$$U = P^\nu \left( \frac{T_g - T_s}{T_g - T_{s_0}} \cdot \frac{T_{s_0} - T_0 - Q_s/c}{T_s - T_N - Q_s/c} \right)^{1/2}$$

or

$$P^\nu = U \left( \frac{B - (T_N - T_0)}{B} \cdot \frac{1 + \gamma D \ln U}{1 - \gamma \eta \ln U} \right)^{1/2}. \quad (30)$$

Here,  $U=u/u_0$ ,  $P=p/p_0$ ,  $B=T_{s_0} - T_0 - Q_s/c$ ,  $D = \frac{\theta + \sigma}{1 - \sigma} = \frac{T_N + Q_s/c}{T_{s_0} - T_N - Q_s/c}$ .

The equation (30) represents a stationary combustion velocity law that takes into account the variability of the combustion surface temperature. When  $\gamma=0$ , this law transforms into the power-law combustion velocity law (29). The parameter  $B$  determines the temperature dependence of the combustion velocity, but when  $\gamma \neq 0$ , it should be noted that the magnitude of  $D$  also depends on the initial fuel temperature. If  $u_0$  denotes the stationary combustion velocity at the fuel temperature  $T_N$  rather than the standard temperature  $T_0$ , then from the equation  $Q=V$ , the stationary combustion velocity law can be expressed as follows:

$$P^\nu = U \left( \frac{1 + \gamma D \ln U}{1 - \gamma \eta \ln U} \right)^{1/2}, \quad (31)$$

The expression (31) can be derived from (30) when  $T_0=T_N$ .

Equation (30) can be used to find the values of  $\nu, Q_s, E_s, T_{s_0}, T_g$  based on the determination of the burning rate under steady-state conditions in a constant pressure apparatus (CPA). Let there be  $n$  one-dimensional arrays  $U_{ij}, P_{ij}$  ( $i=1,2,\dots,m, j=1,2,\dots,n$ ) of experimental data for determining CPA values of the burning rate  $U_{ij}$  at pressures  $P_{ij}$  and fuel temperatures  $T_j$ . Let's first set  $P=1$ , i.e., consider the dependence of the burning rate on the initial fuel temperature at standard pressure. Equation (30) will take the form:

$$B(U_j^2 - 1) + B\gamma\eta \ln U_j + \gamma(T_{s_0} - B)U_j^2 \ln U_j + \gamma U_j^2 \ln U_j \alpha_j = U_j^2 \alpha_j \quad (32)$$

Here,  $U_j$  burning rate at temperature  $T_j, \alpha_j = T_j - T_0, U_j, \alpha_j$  - known values,  $B, C=B\gamma\eta$  and  $D=\gamma(T_{s_0} - B)$  - unknown values. Finding  $B, C$ , and  $D$  using the least squares method and neglecting the small quantity  $\gamma U_j^2 \ln U_j \alpha_j$ , we obtain a system of equations:

$$\begin{cases} B \sum_{j=1}^n (U_j^2 - 1)^2 + C \sum_{j=1}^n (U_j^2 - 1) \ln U_j + D \sum_{j=1}^n U_j^2 (U_j^2 - 1) \ln U_j = \sum_{j=1}^n \alpha_j U_j^2 (U_j^2 - 1), \\ B \sum_{j=1}^n (U_j^2 - 1) \ln U_j + C \sum_{j=1}^n \ln^2 U_j + D \sum_{j=1}^n U_j^2 \ln^2 U_j = \sum_{j=1}^n \alpha_j U_j^2 \ln U_j, \\ B \sum_{j=1}^n (U_j^2 - 1) U_j^2 \ln U_j + C \sum_{j=1}^n U_j^2 \ln^2 U_j + D \sum_{j=1}^n U_j^4 \ln^2 U_j = \sum_{j=1}^n \alpha_j U_j^2 \ln U_j. \end{cases} \quad (33)$$

Now let's assume  $T_N=const$ , i.e., we consider the dependence of the burning rate on pressure at constant fuel temperature. After logarithmization and transformations, equation (30) takes the form:"

$$\ln U_i \left[ 1 + \frac{\gamma}{2} \left( \frac{T_{s_0}}{B - \alpha} + \eta - 1 \right) \right] + \frac{1}{2} \ln \left( 1 - \frac{\alpha}{B} \right) = \nu \ln P_i \quad (34)$$

The quantity  $B$  is known from solving the previous system of equations,  $\alpha$  is constant, and the values  $\alpha, U_i$  and  $P_i$  are known. Let's apply the least squares method to equation (34):

$$\begin{cases} \nu \sum_{i=1}^m \ln P_i \ln U_i - A \sum_{i=1}^m \ln^2 U_i = \beta \sum_{i=1}^m \ln U_i, \\ \nu \sum_{i=1}^m \ln^2 P_i - A \sum_{i=1}^m \ln P_i \ln U_i = \beta \sum_{i=1}^m \ln P_i. \end{cases} \quad (35)$$

Here,  $\beta = \frac{1}{2} \ln \left( 1 - \frac{\alpha}{B} \right)$ ,  $A = 1 + \frac{\gamma}{2} \left( \frac{T_{s_0}}{B - \alpha} + \eta - 1 \right)$ . After obtaining the values of  $B, C, D, \nu, A$  from the systems (33) and (35), it is possible to find  $\gamma, \eta, T_{s_0}, T_g, E_s, Q_s$  using the following formulas:

$$\begin{aligned} \gamma &= \frac{B - \alpha}{\alpha} \left( 2A - \frac{C}{B} + \frac{D}{B - \alpha} - 2 \right), & \eta &= \frac{C}{\gamma B}, \\ T_{s_0} &= B + \frac{D}{\gamma}, & Q_s &= c \left( \frac{D}{\gamma} - T_0 \right), \\ E_s &= \gamma R T_{s_0}, & T_g &= T_{s_0} \frac{C}{C - \gamma B}. \end{aligned} \quad (36)$$

The obtained dependencies completely solve the posed problem. The theory of stationary combustion of solid fuel has been considered as a special case outlined before of the theory of stationary combustion. This general position, in constructing a physico-mathematical model of stationary combustion of solid fuel, is obtained by an equation for calculating the stationary combustion velocity. The theoretical power-law of combustion velocity is introduced. For the first time, the stationary combustion velocity of solid fuel containing a combustion catalyst in its composition is theoretically investigated. A mechanism of interaction between free radicals and metals in a paramagnetic state is proposed. A single combustion velocity law is introduced for a wide range of pressures for fuel containing a catalyst. The method for calculating coefficients from the law of steady-state combustion in the presence of catalysts without fuel composition is given. The method for calculating coefficients from the law of steady-state combustion in the presence of catalysts without fuel composition is given. Let's propose a method for calculating constants that enter into the integral equation for the combustion rate. Implications and Applications: Understanding the stationary combustion velocity in the presence of catalysts has significant implications for various fields, including combustion engineering, energy production, and environmental science. The developed method for calculating constants can be applied to optimize combustion processes, improve fuel efficiency, and reduce emissions in practical applications.

Summarize the key findings of the study and underscore the importance of understanding and quantifying the influence of catalysts on stationary combustion velocity. This research contributes to the broader goal of enhancing combustion efficiency and reducing environmental impact in various industrial sectors.

This research contributes to the understanding of the intricate relationship between catalysts and stationary combustion velocity. The proposed method for calculating constants provides a valuable tool for quantifying the impact of catalysts on combustion dynamics, offering insights that can be leveraged for practical applications in the fields of energy and environmental engineering.

### The Analytical Solution of the Linearized Integral Equation for the Burning Rate

The analytical solution of the linearized integral equation for the burning rate involves solving a mathematical expression that describes the rate at which a substance undergoes combustion. In combustion studies, the burning rate is a crucial parameter that characterizes the combustion process. The linearized integral equation is often employed to model and analyze the behavior of the burning rate under certain conditions.

To develop the analytical solution, one typically begins by linearizing the integral equation, which involves simplifying the equation by assuming small perturbations around a steady state. This linearization process facilitates the application of mathematical techniques to derive a solution that describes the behavior of the burning rate in a more tractable form.

The solution to the linearized integral equation is obtained by solving the resulting linear equations or integrals. Depending on the specific form of the integral equation and the underlying assumptions, various mathematical methods such as Laplace transforms, Fourier transforms, or other specialized techniques may be applied.

It's important to note that the specifics of the analytical solution depend on the form of the integral equation used to model the burning rate and the assumptions made during the linearization process. The solution may involve mathematical functions, constants, and parameters that are relevant to the specific combustion scenario being studied.

In summary, the analytical solution of the linearized integral equation for the burning rate is a mathematical expression that describes the combustion process under simplified conditions. The solution provides insights into the behavior of the burning rate and is valuable in the field of combustion science and engineering.

The analytical solution for the burning rate in combustion problems is often derived from simplified models and assumptions. The linearized integral equation for the burning rate is typically associated with the study of combustion waves or flame propagation. One common approach is to use the Zeldovich-Novozhilov model, which provides a simplified description of the flame structure.

One direction of research into the integral equation for the burning rate of solid fuel involves linearization, i.e., the study of small perturbations in the burning rate with small deviations in pressure from the initial value. Linearization allows obtaining an analytical solution. Let's assume that

$$U(t) = 1 + \alpha(t), \quad P(t) = 1 + \beta(t), \quad y(t) = \int_0^t \alpha(t) dt,$$

$$Y(t) = \int_0^t U(t) dt = t + y(t),$$

while doing so,  $\alpha^2 \ll 1$ ,  $\beta^2 \ll 1$ ,  $y^2 \ll 1$ , so, from (26) the linearized equation should be derived

$$\int_0^t \left( D\alpha + ny - (A\beta + ny - B\alpha) \operatorname{erf} \left( \frac{\sqrt{n(t-\tau)}}{2} \right) \right) d\tau = 0 \quad (37)$$

Here,  $A=2\nu(1-\sigma)$ ,  $B=1-2\nu+\gamma(1-\sigma)(\eta-1)$ ,  $D=\gamma(1+\theta)$ . Equation (37) is a linear integral equation of the Volterra type with a kernel that depends only on the difference of the arguments. After applying the Laplace transform to this equation, we obtain.

$$L_{\alpha}(s) = \frac{A \cdot L_{\beta}(s) \cdot s/n}{\sqrt{1+4 \cdot s/n} (1+D \cdot s/n) + B \cdot s/n - 1} \quad (38)$$

Here,  $L_{\alpha}(s) = \int_0^{\infty} \alpha(t)e^{-ts}$  is the Laplace image of function  $\alpha(t)$ , similarly  $L_{\beta}(s)$  is the Laplace image of function  $\beta(t)$ . Function  $\beta(t)$  we consider known, and the value

$$L_M(s) = \frac{A \cdot s/n}{\sqrt{1+4 \cdot s/n} (1+D \cdot s/n) + B \cdot s/n - 1} \quad (39)$$

we will consider an image by Laplace of a certain as yet unknown function  $M(t)$ . From (38) and the theorem of image multiplication, it follows:

$$\alpha(t) = \int_0^t M(\tau)\beta(t-\tau)d\tau. \quad (40)$$

Let's find the function  $M(t)$  from its image (39). Let  $x = \sqrt{4 \cdot s/n + 1}$ , then after transformations from (39), we obtain:

$$L_M(x) = \frac{A(x+1)}{Dx^2 + (B+D)x + B+4} \quad (41)$$

or after expanding (41) into simple fractions

$$L_M(x) = \frac{A}{D(b-a)} \left( \frac{1-a}{x+a} + \frac{b-1}{x+b} \right), \quad (42)$$

where

$$a = \frac{D+B}{2D}(1-\Delta), \quad b = \frac{D+B}{2D}(1+\Delta),$$

$$\Delta^2 = 1 - \frac{4D(B+4)}{(D+B)^2}.$$

It can be shown that if  $L_f(s) = \frac{1}{\sqrt{1+4 \cdot s/n} + K}$  is the Laplace transform of the function  $f(t)$ , then

$$f(t) = \frac{n}{4} e^{-nt/4} \left( \frac{2}{\sqrt{\pi nt}} - K e^{K^2 nt/4} \cdot \operatorname{erfc} \left( K \sqrt{nt}/2 \right) \right), \quad (43)$$

thus, from (40), (42), and (43), it follows

$$M[z(t)] = \frac{An}{4D(b-a)} \left( \frac{e^{z^2}}{\sqrt{\pi z}} - a(1-a)e^{-(1-a^2)z^2} \operatorname{erfc}(az) + b(1-b)e^{-(1-b^2)z^2} \operatorname{erfc}(bz) \right) \quad (44)$$

and

$$U(t) = 1 + \frac{An}{4D(b-a)} \cdot \int_0^t \beta(t-\tau) \cdot \left( \frac{e^{-z^2}}{\sqrt{\pi-z}} - a(1-a)e^{-(1-a^2)z^2} \operatorname{erfc}(az) + b(1-b)e^{-(1-b^2)z^2} \operatorname{erfc}(bz) \right) d\tau, \quad (45)$$

where  $z(t) = \sqrt{nt}/2$ . The formula (45) provides a complete solution to the problem of calculating the nonstationary combustion velocity in a general form with the sole restriction that  $\beta^2 \ll 1$ .

Let's denote  $\delta = 4D(B+4)/(D+B)^2$ . If  $\delta > 1$ , the quantity  $\Delta$  becomes imaginary, and consequently, the coefficients  $a$  and  $b$  become complex conjugates. In this case, the calculation according to formula (45) is complicated by the need to compute functions like  $\operatorname{erfc}(W)$  with a complex argument  $W = x + iy$ . This difficulty can be overcome by introducing special functions of two real variables,  $x$  and  $y$  –  $\operatorname{Se}(x,y)$  and  $\operatorname{Ce}(x,y)$  such that

$$\operatorname{erfc}(x + iy) = Se(x, y) - iCe(x, y). \quad (46)$$

From the definition  $\operatorname{erfc}(W) = 1 - \frac{2}{\sqrt{\pi}} \int_0^W e^{-t^2} dt$  after substituting

the integration variable  $t = x + si$  and performing the necessary transformations, we obtain:

$$\operatorname{erfc}(x + iy) = \operatorname{erfc}(x) - \frac{2e^{-x^2}}{\sqrt{\pi}} \int_0^y e^{s^2} \sin(2sx) ds - i \frac{2e^{-x^2}}{\sqrt{\pi}} \int_0^y e^{s^2} \cos(2sx) ds. \quad (47)$$

Then, from (46)

$$\begin{cases} Se(x, y) = \operatorname{erf}(x) - \frac{e^{-x^2}}{\sqrt{\pi}} \cdot x \cdot y \sum_{n=0}^{\infty} \left( \frac{y^{2n}}{n+1} \sum_{j=0}^n \frac{x^{2j} (-1)^j}{(n-j)!(2j+1)!} \right), \\ Ce(x, y) = \frac{2e^{-x^2}}{\sqrt{\pi}} \cdot y \sum_{n=0}^{\infty} \frac{y^{2n}}{2n+1} \sum_{j=0}^n \frac{x^{2j} (-1)^j}{(n-j)!(2j)!}. \end{cases} \quad (48)$$

Formulas (48) are derived from (47) by expanding the sub-integral functions into series with term-wise integration. Substituting the values of coefficients a and b into (45) when  $\delta > 1$ , and using the relationship (45) along with the formula for the exponential function of a complex argument, we can bring this formula to the following form:

$$U(t) = 1 + \frac{An}{4D} \int_0^t \beta(t-\tau) \left( \frac{e^{-z^2}}{\sqrt{\pi z}} + F(z) e^{Nz^2} \cos(\omega z^2 + \phi(z)) \right) d\tau. \quad (49)$$

Here

$$F(z) = 2 \sqrt{\frac{Se^2(x, y) + Ce^2(x, y)}{D(\delta - 1)}},$$

$$\phi(z) = \arctan \frac{(B^2 - BD - 8D)Se(x, y) + B\sqrt{\delta - 1}(B + D)Ce(x, y)}{(B^2 - BD - 8D)Ce(x, y) - B\sqrt{\delta - 1}(B + D)Se(x, y)},$$

$$N = -(D^2 + 8D - B^2) / (2D^2), \quad \omega = (B + D)^2 \sqrt{\delta - 1} / (2D^2),$$

$$x = (B + D)z / (2D), \quad y = (B + D)\sqrt{\delta - 1} \cdot z / (2D), \quad z = \sqrt{nt}/2.$$

The formulas (45) or (49) provide an analytical solution to the linearized integral equation for the unsteady combustion velocity at arbitrary, but small deviations of pressure from the initial value. It's important to note that the solution above is a simplified model, and in practical situations, combustion problems are often more complex. The solution's accuracy depends on the specific assumptions made during its derivation.

### Particular Solutions of the Linearized Integral Equation

An integral equation is an equation in which an unknown function appears under an integral sign. Linearizing an integral equation typically involves approximating it in a way that makes it linear or easier to handle mathematically. This often involves linearizing around a specific point or operating in the neighborhood of a solution.

Particular Solutions: Particular solutions refer to specific solutions that satisfy the given equation. These solutions are often found by applying specific conditions or constraints to the equation.

Let's consider the case of stepwise pressure changes:

$$P(t) = \begin{cases} 1 & \text{if } t = 0, \\ 1 + \beta & \text{if } t > 0. \end{cases}$$

In this case  $\beta(t) = \beta = \text{const}$  and from (45) follows

$$U(t) = 1 + \beta \int_0^t M(\tau) d\tau.$$

After integrating expression (45), we obtain

$$U(t) = 1 + \frac{A \cdot \beta}{D(a+1)(b+1)} \times \left( 1 + \operatorname{erf}(z) + \frac{a(b+1)}{b-a} e^{-z^2(1-a^2)} \operatorname{erfc}(az) - \frac{b(a+1)}{b-a} e^{-z^2(1-b^2)} \operatorname{erfc}(bz) \right) \quad (50)$$

Here  $a = \frac{D+B}{2D} (1 + \sqrt{1-\delta})$ ,  $b = \frac{D+B}{2D} (1 - \sqrt{1-\delta})$ ,  $\delta = \frac{4D(b+4)}{(D+B)^2} < 1$ ,  $z = \sqrt{nt}/2$ .

To find  $U(t)$  when  $\delta > 1$ , substitute the coefficients into equation (50) a and b in the form  $a = m + pi$ ,  $b = m - pi$ ,  $m = (D + B)/(2D)$ ,

$p = m\sqrt{\delta - 1}$ . Using the formulas obtained earlier for

calculating  $\operatorname{erfc}(az)$  and  $\operatorname{erfc}(bz)$ , as well as the formula for the exponential of a complex argument, we obtain

$$U(t) = 1 + \frac{A \cdot \beta}{D((m+1)^2 + p^2)} (1 + \operatorname{erf}(z) + Am(z) \cos(\omega z^2 + \phi(z))). \quad (51)$$

Here

$$\begin{aligned} Am(z) &= \frac{e^{Nz^2}}{p} \sqrt{(Q^2 + P^2) \cdot (Se^2(x, y) + Ce^2(x, y))} && \text{— amplitude of eigenvalues} \\ & && \text{oscillations of combustion velocity;} \\ \phi(z) &= \arctan \frac{Q \cdot Se(x, y) + p \cdot Ce(x, y)}{Q \cdot Ce(x, y) - p \cdot Se(x, y)} && \text{— phase of eigenmode oscillations;} \\ \omega &= 2mp && \text{— natural frequency of oscillations,} \end{aligned}$$

$x = mz$ ,  $y = pz$ ,  $Q = m^2 + p^2 + m$ ,  $N = m2 - p^2 - 1$ .

Now let's consider the pressure as a function of time in the form:

$$P(t) = \begin{cases} 1 + \beta t & \text{if } t \leq t_0, \\ 1 + \beta t_0 & \text{if } t > t_0. \end{cases} \quad (52)$$

This dependence describes the transition of pressure to a new steady-state mode, where  $\beta$  is a constant representing the pressure gradient. In this case, integrating formula (45) is quite challenging; however, it is possible to derive the formula for the burning velocity directly from the dependence (38). Taking into account that

$L_{\beta}(s) = \beta(1 - e^{-st_0})/s^2$ , applying the convolution

theorem to (38) yields

$$U(t) = \begin{cases} 1 + M(t) & \text{if } t \leq t_0, \\ 1 + M(t) - M(t - t_0) & \text{if } t > t_0, \end{cases} \quad (53)$$

where  $M(t)$  – a function with an image

$$L_M(x) = \frac{4A\beta}{Dn} \cdot \frac{1}{(x-1)^2(x+1)(x+a)(x+b)}, \quad (54)$$

where  $x = \sqrt{4s/n + 1}$ . After expanding (54) into partial fractions (using the Vashchenko-Zakharchenko theorem) when  $\delta < 1$ , upon transitioning from the transform (54) back to the original function, the function  $M(t)$  will take the form

$$M[z(t)] = \frac{\beta \cdot A}{D} \left( c_1 z \left( z + \frac{e^{-z^2}}{\sqrt{\pi}} \right) + (c_1 z^2 + c_2) \operatorname{erf}(z) + c_3 e^{-z^2(1-a^2)} (\operatorname{erfc}(az) - 1) - c_4 e^{-z^2(1-b^2)} (\operatorname{erfc}(bz) - 1) \right). \quad (55)$$

Here

$$\begin{aligned} c_1 &= \frac{1}{(a+1)(b+1)}, & c_2 &= \frac{(a^2+1)(b^2+1) + 2(a+1)(b+1) - 4}{2(a^2-1)(b^2-1)(a+1)(b+1)}, \\ c_3 &= \frac{a}{(a^2-1)(a+1)(b-a)}, & c_4 &= \frac{b}{(b^2-1)(a+1)(b-a)}. \end{aligned}$$

When  $\delta > 1$ , substituting  $a = m + \pi$  and  $b = m - \pi$  into formula (55) yields

$$M[z(t)] = \frac{A\beta}{D} \left( c_1 z \left( z + \frac{e^{-z^2}}{\sqrt{\pi}} \right) + (c_1 z^2 + c_2) \operatorname{erf}(z) + Am(z) \cos(\omega z^2 + \phi(z)) \right). \quad (56)$$

Here

$$Am(z) = \frac{e^{Nz^2}}{Gp} \sqrt{Q^2 + R^2} \cdot \sqrt{Se^2(x, y) + Ce^2(x, y)},$$



$$\phi(z) = \arctan \frac{Q \cdot Fe(x, y) + R \cdot Ce(x, y)}{Q \cdot Ce(x, y) - R \cdot Se(x, y)},$$

$$Q = (m^2 + m + p^2) \cdot N, \quad R = p(N - 2m(m^2 + p^2 + m)),$$

$$G = ((m + 1)^2 + p^2)^2 ((m - 1)^2 + p^2), \quad \omega = 2mp, \quad N = m^2 - p^2 - 1.$$

Using the same method, one can obtain an analytical solution for the pressure as a function of time in the form of

$$P(t) = \beta \sin kt \quad (57)$$

Here,  $\beta$  is a constant, and  $k$  is the frequency of pressure oscillations. Without deriving it here, let's present the final formula for  $\delta < 1$

$$U(t) = 1 + A_0 \left[ A_1 \sin(kt + \phi_1) + A_2(t) \sin(kt + \phi_2 + \phi_3(t)) + A_3 e^{-z^2(1-a^2)} \operatorname{erfc}(az) + A_4 e^{-z^2(1-b^2)} \operatorname{erfc}(bz) \right] \quad (58)$$

$$\phi_1 = \arctan(c_1/c_4), \quad \phi_2 = \arctan(c_2/c_3), \quad \phi_3(t) = \arctan \frac{Ce(x, y)}{Se(x, y)},$$

$$A_1^2 = c_1^2 + c_4^2, \quad A_2^2(t) = (c_3^2 + c_3^2) (Se^2(x, y) + Ce^2(x, y)),$$

$$A_0 = 16 \cdot A\beta k / (Dn^2), \quad r^2 = 1 + (4k/n)^2, \quad m^2 = (r + 1)/2,$$

$$p^2 = (r - 1)/2, \quad x = mz, \quad y = pz.$$

The constants  $c_1$ - $c_4$  expressed through  $a, b, k, r, m, p$ . It is also possible to obtain an analytical solution for  $\delta > 1$  and a separate solution for the resonance case (matching the frequencies of the eigenmode oscillations of the burning velocity and the pressure oscillations, i.e., when  $\omega = k$ ). However, the investigation of these processes is beyond the scope of the current study.

### Numerical and Approximate Solutions of the Nonlinear Integral Equation for the Burning Velocity

The burning velocity in combustion processes is often described by a nonlinear integral equation. This equation typically involves complex interactions and dependencies on various parameters. Numerical methods are commonly employed to approximate the solution to such nonlinear integral equations. Finite difference methods, finite element methods, and spectral methods are often applied in the discretization of the integral term.

The nonlinear integral equation for the combustion velocity (26) apparently does not have an exact analytical solution. However, for calculating the combustion velocity based on this equation, both approximate-analytical solutions and numerical methods, such as the finite sum method can be applied [40]. The application of this method allows expressing equation (26) as follows:

$$\sum_{i=1}^n A_i ((U_i Q_i - E_i) \operatorname{erf}(\psi_{ij}) - U_i (V_i - E_i)) = 0. \quad (59)$$

Here,

$$\psi_{ij} = \begin{cases} \frac{\sqrt{nh}}{2} \left( \sum_{i=1}^n A_i U_i - \sum_{j=1}^i A_j U_j \right) / \sqrt{n-i} & \text{if } i \neq n, \\ 0 & \text{if } i = n. \end{cases}$$

$A_i, A_j$  – numerical coefficients in the formulas representing integrals by finite sums, for example, in the Simpson's rule formula, ( $n = 2m + 1$ )  $A_1 = A_{2m+1} = 1/3, A_2 = A_4 = \dots = A_{2m} = 4/3, A_3 = A_5 = \dots = A_{2m-1} = 2/3, U_j, U_i, Q_i, V_i, E_i$  – values of variables  $U, Q, V, E$  in moments  $\tau = \tau_j$  or  $\tau = \tau_i$ , where  $i = 1, 2, \dots, n, j = 1, 2, \dots, i$  – numbers of the points of partitioning of segments  $[0, t]$  or  $[0, \tau_i]$  respectively,  $h = t/(n - 1)$  – step of segment partitioning  $[0, t]$  by

points  $\tau_i$ . If all values of the burning rate are known on the interval  $[0, t - h]$  (they can be calculated, for example, using the formulas of linear theory), then equation (59) includes only one unknown  $U_n = U(t)$ . In this case, equation (59) is a nonlinear algebraic equation with respect to  $U_n$ . After calculating  $U_n$  as the root of this equation, the process can be continued using this value of  $U_n$  to find  $U_{n+1}, U_{n+2}$ , and so on. However, it should be noted that an effective algorithm for computing the root of equation (59) has not yet been developed due to the complex (oscillatory) behavior of the function  $U(t)$ .

To calculate the combustion speed according to equation (59), an approximate analytical method has also been developed. Let the pressure and combustion speed values satisfy the inequalities until the moment  $t = t_1$ :

$$\left( \frac{p(t)}{p_0} - 1 \right)^2 \ll 1, \quad \left( \frac{u(t)}{u_0} - 1 \right)^2 \ll 1, \quad (60)$$

what gives the opportunity to calculate  $U(t)$  according to the formulas of linear theory, i.e. we will consider functions  $U(t), V(t), Q(P, U)$  in the interval  $[0, t_1]$  known. We will designate  $U(t_1) = U_1, T_s(t_1) = T_{s1}$ . Equalities are valid:

$$T_{s1} = T_N + (T_{s0} - T_N)V(U_1),$$

$$\varepsilon(t_1) = \frac{\varepsilon}{u_0} (T_{s0} - T_N) \left( 1 + n \int_0^{t_1} U(V - Q) d\tau \right) \quad (61)$$

The quantity  $\varepsilon(t)$  is proportional to the amount of thermal energy in the  $k$ -phase and depends on the temperature distribution in the  $k$ -phase:

$$\varepsilon(t) = \int_0^\infty (T(x, t) - T_N) dx,$$

moreover

$$\varepsilon(0) = \frac{\varepsilon}{u_0} (T_{s0} - T_N).$$

Let's assume that the actual temperature distribution in the  $k$ -phase at the moment  $t = t_1$  can be approximated by the Michelson dependence in the same form as the dependence (10) adopted by us as the initial condition, i.e., at the moment  $t = t_1$ :

$$T(x, t_1) = T_N + (T_{s1} - T_N) e^{-\frac{u_1}{\sigma} x}, \quad \varepsilon(t_1) = \frac{\varepsilon}{u_1} (T_{s1} - T_N). \quad (62)$$

Equating values  $\varepsilon(t_1)$  from (61) and (62) we get an equation

$$n_1 = \frac{n U_1 V(U_1)}{1 + n \int_0^{t_1} U(V - Q) d\tau} \quad (63)$$

Now we can again apply the formulas of linear theory, assuming the moment  $t = t_1$  for the initial one and recalculating the coefficients  $n, \sigma, \theta, \gamma, \eta$  taking into account the new initial values  $u_0 = u_1$  and  $T_{s0} = T_{s1}$ . In addition, when calculating the function  $E(t)$ , it is necessary to replace the coefficient  $n$  with the coefficient  $n_1$ .

This approach can be applied several times; however, it should be kept in mind that the approximation of the temperature distribution in the  $k$ -phase by the Michelson distribution is approximate. This introduces some error into the calculated values of  $U(t)$ , the acceptability of which needs to be justified.

A linear approximation for the main equation describing the combustion velocity of solid fuel has been derived. The obtained equation is a first-order Volterra linear integral equation of convolution type. Using Laplace transformation, a general analytical solution for the linear approximation of the main equation for steady combustion velocity has been obtained. It is demonstrated that the solution for the linear approximation of the main equation for combustion velocity can be expressed as a linear integral operator of the Volterra convolution type. The solution for the linear approximation of the main equation for combustion velocity is obtained for step, smooth, and oscillatory changes in length.

The description outlines an approximate-analytical method for solving the main equation of non-stationary combustion velocity, considering a specified pressure variation. The ballistic characteristics of a specific solid rocket motor have been calculated for the first time using formulas from non-stationary fuel combustion theory. Short-term agreements between calculated and experimental data have been obtained.

The nonlinear integral equation governing burning velocity presents challenges in obtaining exact solutions. Therefore, a combination of numerical and approximate methods is often employed to efficiently and accurately approximate the burning velocity under various conditions.

The choice of an effective method for the numerical solution of the nonlinear integral equation for combustion rate is one of the tasks for further research.

## Conclusions

Now, let's enumerate some important conclusions from this work, namely:

- This paper is a continuation of [41,42]. A physical-mathematical model of solid fuel combustion has been developed, aimed at solving the main problem of internal ballistics - calculating the pressure in the combustion chamber of a solid rocket motor (SRM).
- The developed physical-mathematical model of solid fuel combustion includes:
  - a. Heat propagation equation in the condensed phase taking into account heat release on the combustion surface;
  - b. Arrhenius equation for the burning rate as a function of the surface temperature of combustion;
  - c. Equation for the heat flux from the gas phase to the condensed phase, based on Semenov's equation for the induction period in the course of chain-branching chemical reactions.
- Based on the developed physical-mathematical model, a nonlinear integral equation has been obtained for calculating the non-stationary burning rate of solid fuel.
- The stationary solution of the integral equation has been obtained—the refined law of the stationary burning rate of fuel. A method for calculating the parameters of the refined burning rate law based on the determination of the burning rate under stationary conditions has been proposed.
- An approximate (linearized) integral equation for the burning rate of solid fuel has been obtained. Its general solution and particular solutions for cases of step, smooth, and oscillatory pressure variations have been found.
- Preliminary studies have been conducted on the development of a numerical solution method for the linear integral equation for burning rate.
- We have obtained an approximate (linearized) integral equation for the velocity.

## Recommendations

To implement the results of this study into the practice of pulse detonation engine development it is recommended to continue research in the following directions:

- Conduct processing of the results of determining the burning rate in steady-state conditions for specific solid fuel compositions in accordance with the methodology outlined in section 3.1, with the aim of determining the coefficients of the refined combustion law and the coefficients involved in the equation for the non-stationary burning rate. Compile a system of equations for the calculation of the PDE (Pulse Detonation Engine) combustion chamber, including the nonlinear integral equation for the non-stationary burning rate obtained before.
- Develop effective numerical methods for solving the formulated system of equations based on the methods obtained before, and create calculation programs. Conduct calculations for specific pulse-jet rocket engines and compare the calculation results with experimental data.

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